



Introduction

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1.1 INTRODUCTION

An open channel is a conduit in which a liquid flows with a free surface. The free surface is actually an interface between the moving liquid and an overlying fluid medium and will have constant pressure. In civil engineering applications; water is the most common liquid with air at atmospheric pressure as the overlying fluid. As such, our attention will be chiefly focused on the flow of water with a free surface. The prime motivating force for open channel flow is gravity.

In engineering practice, activities for utilization of water resources involve open channels of varying magnitudes in one way or the other. Flows in natural rivers, streams and rivulets; artificial, i.e. man-made canals for transmitting water from a source to a place of need, such as for irrigation, water supply and hydropower generation; sewers that carry domestic or industrial waste waters; navigation channels—are all examples of open channels in their diverse roles. It is evident that the size, shape and roughness of open channels vary over a sizeable range, covering a few orders of magnitude. Thus the flow in a road side gutter, flow of water in an irrigation canal and flows in the mighty rivers, such as the Ganga and the Brahmaputra, all have a free surface and as such are open channels, governed by the same general laws of fluid mechanics. Basically, all open channels have a bottom slope and the mechanism of flow is akin to the movement of a mass down an inclined plane due to gravity. The component of the weight of the liquid along the slope acts as the driving force. The boundary resistance at the perimeter acts as the resisting force. Water flow in open channels is largely in the turbulent regime with negligible surface tension effects. In addition, the fact that water behaves as an incompressible fluid leads one to appreciate the importance of the force due to gravity as the major force and the Froude number as the prime non-dimensional number governing the flow phenomenon in open channels.

1.2 TYPES OF CHANNELS

1.2.1 Prismatic and Non-prismatic Channels

A channel in which the cross-sectional shape and size and also the bottom slope are constant is termed as a prismatic channel. Most of the man-made (artificial) channels are prismatic channels over long stretches. The rectangle, trapezoid, triangle

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and circle are some of the commonly used shapes in manmade channels. All natural channels generally have varying cross-sections and consequently are non-prismatic.

1.2.2 Rigid and Mobile Boundary Channels

On the basis of the nature of the boundary open channels can be broadly classified into two types: (i) rigid channels, and (ii) mobile boundary channels.

Rigid channels are those in which the boundary is not deformable in the sense that the shape, planiform and roughness magnitudes are not functions of the flow parameters. Typical examples include lined canals, sewers and non-erodible unlined canals. The flow velocity and shear-stress distribution will be such that no major scour, erosion or deposition takes place in the channel and the channel geometry and roughness are essentially constant with respect to time. The rigid channels can be considered to have only one degree of freedom; for a given channel geometry the only change that may take place is the depth of flow which may vary with space and time depending upon the nature of the flow. This book is concerned essentially with the study of rigid boundary channels.

In contrast to the above, we have many unlined channels in alluvium—both man-made channels and natural rivers—in which the boundaries undergo deformation due to the continuous process of erosion and deposition due to the flow. The boundary of the channel is mobile in such cases and the flow carries considerable amounts of sediment through suspension and in contact with the bed. Such channels are classified as mobile-boundary channels. The resistance to flow, quantity of sediment transported, channel geometry and planiform, all depend on the interaction of the flow with the channel boundaries. A general mobile-boundary channel can be considered to have four degrees of freedom. For a given channel not only the depth of flow but also the bed width, longitudinal slope and planiform (or layout) of the channel may undergo changes with space and time depending on the type of flow. Mobile-boundary channels, usually treated under the topic of *sediment transport* or *sediment engineering*,^{1,2} attract considerable attention of the hydraulic engineer and their study constitutes a major area of multi-disciplinary interest.

Mobile-boundary channels are dealt briefly in Chapter 11. The discussion in rest of the book is confined to rigid-boundary open channels only. Unless specifically stated, the term *channel* is used in this book to mean the rigid-boundary channels.

1.3 CLASSIFICATION OF FLOWS

1.3.1 Steady and Unsteady Flows

A steady flow occurs when the flow properties, such as the depth or discharge at a section do not change with time. As a corollary, if the depth or discharge changes with time the flow is termed *unsteady*.

In practical applications, due to the turbulent nature of the flow and also due to the interaction of various forces, such as wind, surface tension, etc., at the surface there will always be some fluctuations of the flow properties with respect to time. To

account for these, the definition of steady flow is somewhat generalised and the classification is done on the basis of gross characteristics of the flow. Thus, for example, if there are ripples resulting in small fluctuations of depth in a canal due to wind blowing over the free surface and if the nature of the water-surface profile due to the action of an obstruction is to be studied, the flow is not termed unsteady. In this case, a time average of depth taken over a sufficiently long time interval would indicate a constant depth at a section and as such for the study of gross characteristics the flow would be taken as steady. However, if the characteristics of the ripples were to be studied, certainly an unsteady wave movement at the surface is warranted. Similarly, a depth or discharge slowly varying with respect to time may be approximated for certain calculations to be steady over short time intervals.

Flood flows in rivers and rapidly varying surges in canals are some examples of unsteady flows. Unsteady flows are considerably more difficult to analyse than steady flows. Fortunately, a large number of open channel problems encountered in practice can be treated as steady-state situations to obtain meaningful results. A substantial portion of this book deals with steady-state flows and only a few relatively simple cases of unsteady flow problems are presented in Chapter 10.

1.3.2 Uniform and Non-uniform Flows

If the flow properties, say the depth of flow, in an open channel remain constant along the length of the channel, the flow is said to be *uniform*. As a corollary of this, a flow in which the flow properties vary along the channel is termed as *non-uniform flow* or *varied flow*.

A prismatic channel carrying a certain discharge with a constant velocity is an example of uniform flow [Fig. 1.1(a)]. In this case the depth of flow will be constant along the channel length and hence the free surface will be parallel to the bed. It is easy to see that an unsteady uniform flow is practically impossible, and hence the term uniform flow is used for *steady uniform flow*.

Flow in a non-prismatic channel and flow with varying velocities in a prismatic channel are examples of *varied flow*. Varied flow can be either steady or unsteady.

1.3.3 Gradually Varied and Rapidly Varied Flows

If the change of depth in a varied flow is gradual so that the curvature of streamlines is not excessive, such a flow is said to be a *gradually varied flow* (GVF). Frictional resistance plays an important role in these flows. The backing up of water in a stream due to a dam or drooping of the water surface due to a sudden drop in a canal bed are examples of steady GVF. The passage of a flood wave in a river is a case of unsteady GVF [Fig. 1.1(b)].

If the curvature in a varied flow is large and the depth changes appreciably over short lengths, such a phenomenon is termed as *rapidly varied flow* (RVF). The frictional resistance is relatively insignificant in such cases and it is usual to regard RVF as a local phenomenon. A hydraulic jump occurring below a spillway or a sluice gate is an example of steady RVF. A surge moving up a canal [Fig. 1.1(c)] and a bore traveling up a river are examples of unsteady RVF.

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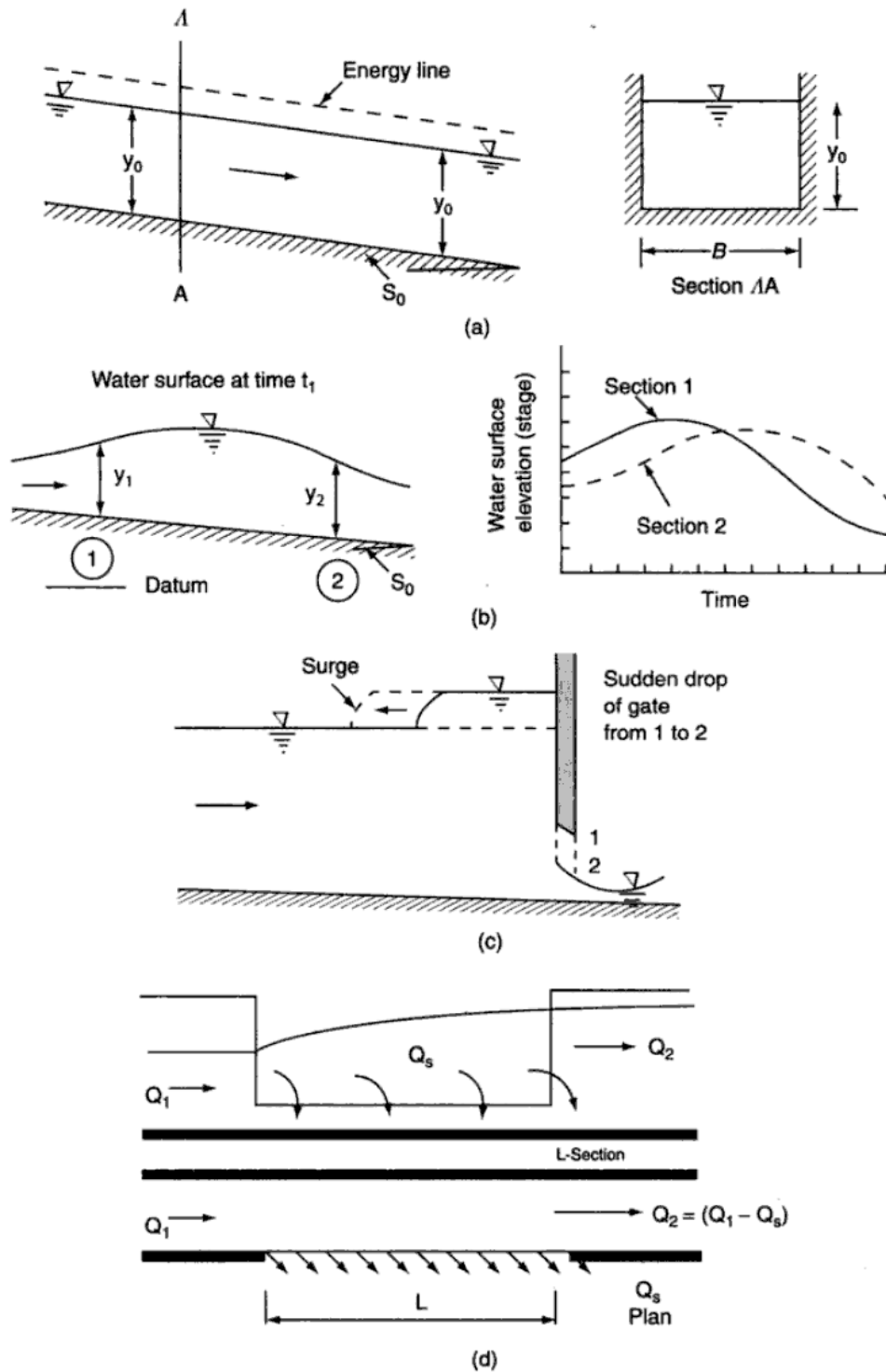


Fig. 1.1 Various types of open channel flows: (a) Uniform flow, (b) Gradually varied flow (c) Rapidly varied flow and (d) SideWeir: Spatially varied flow



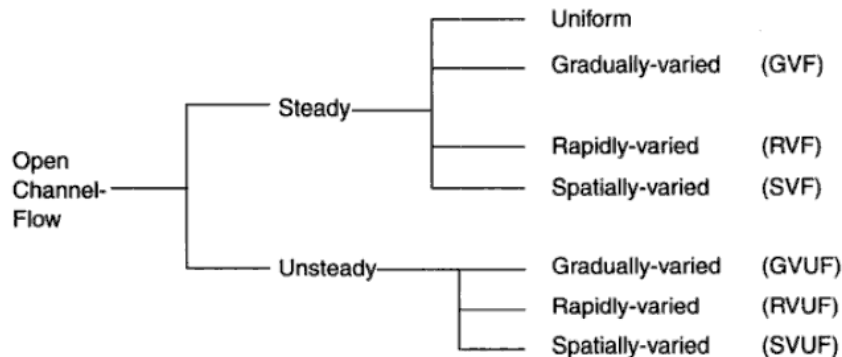
1.3.4 Spatially Varied Flow

Varied flow classified as GVF and RVF assumes that no flow is externally added to or taken out of the canal system. The volume of water in a known time interval is conserved in the channel system. In steady-varied flow the discharge is constant at all sections. However, if some flow is added to or abstracted from the system the resulting varied flow is known as a *spatially varied flow* (SVF).

SVF can be steady or unsteady. In the steady SVF the discharge while being steady-varies along the channel length. The flow over a side weir is an example of steady SVF [Fig. 1.1(d)]. The production of surface runoff due to rainfall, known as overland flow, is a typical example of unsteady SVF.

Classification. Thus open channel flows are classified for purposes of identification and analysis as follows:

Figure 1.1(a) to (d) shows some typical examples of the above types of flows



1.4 VELOCITY DISTRIBUTION

The presence of corners and boundaries in an open channel causes the velocity vectors of the flow to have components not only in the longitudinal and lateral direction but also in normal direction to the flow. In a macro-analysis, one is concerned only with the major component, viz., the longitudinal component, v_x . The other two components being small are ignored and v_x is designated as v . The distribution of v in a channel is dependent on the geometry of the channel. Figure 1.2(a) and (b) show isovels (contours of equal velocity) of v for a natural and rectangular channel respectively. The influence of the channel geometry is apparent. The velocity v is zero at the solid boundaries and gradually increases with distance from the boundary. The maximum velocity of the cross-section occurs at a certain distance below the free surface. This dip of the maximum velocity point, giving surface velocities which are less than the maximum velocity, is due to secondary currents and is a function of the aspect ratio (ratio of depth to width) of the channel. Thus for a deep narrow channel, the location of the maximum velocity point will be much lower from the water surface than for a wider channel of the same depth. This characteristic location

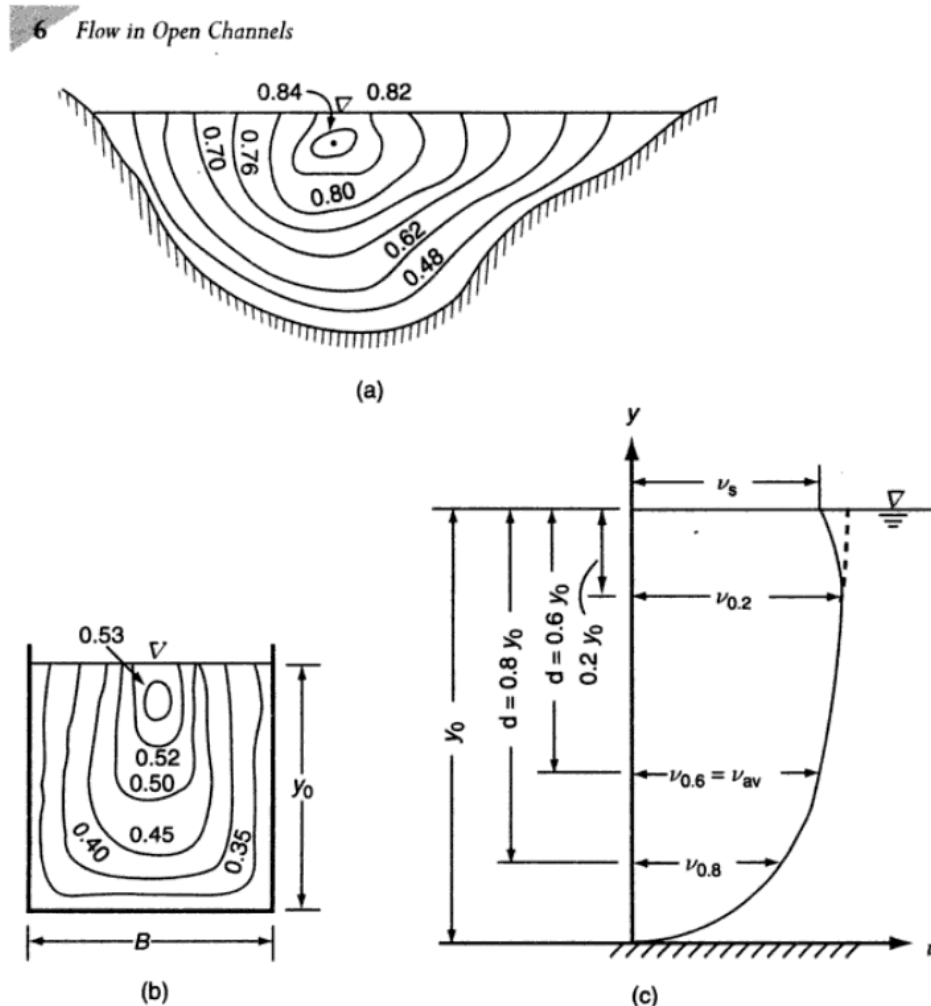


Fig. 1.2 Velocity distribution in open channels: (a) Natural channel (b) Rectangular channel and (c) Typical velocity profile

of the maximum velocity point below the surface has nothing to do with the wind shear on the free surface.

A typical velocity profile at a section in a plane normal to the direction of flow is presented in Fig. 1.2(c). The profile can be roughly described by a logarithmic distribution or a power-law distribution up to the maximum velocity point (Section 3.7). Field observations in rivers and canals have shown that the average velocity at any vertical v_{av} , occurs at a level of $0.6 y_0$ from the free surface, where y_0 = depth of flow. Further, it is found that

$$v_{av} = \frac{v_{0.2} + v_{0.8}}{2} \quad (1.1)$$

in which $v_{0.2}$ = velocity at a depth of $0.2 y_0$ from the free surface, and $v_{0.8}$ = velocity at a depth of $0.8 y_0$ from the free surface. This property of the velocity distribution is



commonly used in stream-gauging practice to determine the discharge using the area-velocity method. The surface velocity v_s is related to the average velocity v_{av} as

$$v_{av} = kv_s \quad (1.2)$$

where, k = a coefficient with a value between 0.8 and 0.95. The proper value of k depends on the channel section and has to be determined by field calibrations. Knowing k , one can estimate the average velocity in an open channel by using floats and other surface velocity measuring devices.

1.5 ONE-DIMENSIONAL METHOD OF FLOW ANALYSIS

Flow properties, such as velocity and pressure gradient in a general open channel flow situation can be expected to have components in the longitudinal as well as in the normal directions. The analysis of such a three-dimensional problem is very complex. However, for the purpose of obtaining engineering solutions, a majority of open channel flow problems are analysed by one-dimensional analysis where only the mean or representative properties of a cross section are considered and their variations in the longitudinal direction is analysed. This method when properly used not only simplifies the problem but also gives meaningful results.

Regarding velocity, a mean velocity V for the entire cross-section is defined on the basis of the longitudinal component of the velocity v as

$$V = \frac{1}{A} \int_A v dA \quad (1.3)$$

This velocity V is used as a representative velocity at a cross-section. The discharge past a section can then be expressed as

$$Q = \int v dA = VA \quad (1.4)$$

The following important features specific to one dimensional open channel flow are to be noted:

- A single elevation represents the water surface perpendicular to the flow.
- Velocities in directions other than the direction of the main axis of flow are not considered.

Kinetic Energy The flux of the kinetic energy flowing past a section can also be expressed in terms of V . But in this case, a correction factor α will be needed as the kinetic energy per unit weight $V^2/2g$ will not be the same as $v^2/2g$ averaged over the cross-section area. An expression for α can be obtained as follows:

For an elemental area dA , the flux of kinetic energy through it is equal to

$$\left(\frac{\text{mass}}{\text{time}} \right) \left(\frac{\text{KE}}{\text{mass}} \right) = (\rho v dA) \frac{v^2}{2}$$

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For the total area, the kinetic energy flux

$$= \int_A \frac{\rho}{2} v^3 dA = \alpha \frac{\rho}{2} V^3 A \quad (1.5)$$

from which

$$\alpha = \frac{\int v^3 dA}{V^3 A} \quad (1.6)$$

or for discrete values of v ,

$$\alpha = \frac{\sum v^3 \Delta A}{V^3 A} \quad (1.7)$$

α is known as the *kinetic energy correction factor* and is equal to or greater than unity.

The kinetic energy per unit weight of fluid can then be written as $\alpha \frac{V^2}{2g}$.

Momentum Similarly, the flux of momentum at a section is also expressed in terms of V and a correction factor β . Considering an elemental area dA , the flux of momentum in the longitudinal direction through this elemental area

$$= \left(\frac{\text{mass}}{\text{time}} \times \text{velocity} \right) = (\rho v dA)(v)$$

For the total area, the momentum flux

$$= \int \rho v^2 dA = \beta \rho V^2 A \quad (1.8)$$

which gives

$$\beta = \frac{\int v^2 dA}{V^2 A} = \frac{\sum v^2 \Delta A}{V^2 A} \quad (1.9)$$

β is known as the *momentum correction factor* and is equal to or greater than unity.

Values of α and β The coefficients α and β are both unity in the case of uniform velocity distribution. For any other velocity distribution $\alpha > \beta > 1.0$. The higher the non-uniformity of velocity distribution, the greater will be the values of the coefficients. Generally, large and deep channels of regular cross sections and with fairly straight alignments exhibit lower values of the coefficients. Conversely, small channels with irregular cross sections contribute to larger values of α and β . A few measured values of α and β are reported by King³. It appears that for straight prismatic channels, α and β are of the order of 1.10 and 1.05 respectively. In compound channels, i.e. channels with one or two flood banks, α and β may, in certain cases reach very high values, of the order of 2.0 (see Compound channels in Sec. 5.7.3).



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Generally, one can assume $\alpha = \beta = 1.0$ when the channels are straight, prismatic and uniform flow or GVF takes place. In local phenomenon, it is desirable to include estimated values of these coefficients in the analysis. For natural channels, the following values of α and β are suggested for practical use⁷:

Channels	Values of α		Values of β	
	Range	Average	Range	Average
Natural channels and torrents	1.15 – 1.50	1.30	1.05 – 1.17	1.10
River valleys, overflowed	1.50 – 2.00	1.75	1.17 – 1.33	1.25

It is usual practice to assume $\alpha = \beta = 1.0$ when no other specific information about the coefficients are available.

Example 1.1

The velocity distribution in a rectangular channel of width B and depth of flow y_0 was approximated as $v = k_1 \sqrt{y}$ in which $k_1 = \text{a constant}$. Calculate the average velocity for the cross section and correction coefficients α and β .

Solution Area of cross section $A = By_0$

$$\begin{aligned} \text{Average velocity } V &= \frac{1}{By_0} \int_0^{y_0} v(B dy) \\ &= \frac{1}{y_0} \int_0^{y_0} k_1 \sqrt{y} dy = \frac{2}{3} k_1 \sqrt{y_0} \end{aligned}$$

Kinetic energy correction factor

$$\alpha = \frac{\int_0^{y_0} v^3 (B dy)}{V^3 B y_0} = \frac{\int_0^{y_0} k_1^3 y^{3/2} B dy}{\left(\frac{2}{3} k_1 \sqrt{y_0}\right)^3 B y_0} = 1.35$$

Momentum correction factor

$$\beta = \frac{\int_0^{y_0} v^2 (B dy)}{V^2 B y_0} = \frac{\int_0^{y_0} k_1^2 y B dy}{\left(\frac{2}{3} k_1 \sqrt{y_0}\right)^2 B y_0} = 1.125$$

Example 1.2

The velocity distribution in an open channel could be approximated as in Fig. 1.3. Determine the kinetic energy correction factor α and momentum correction factor β for this velocity profile.

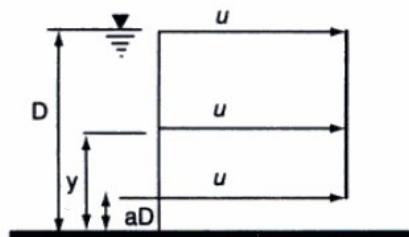
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Fig. 1.3 Velocity distribution of
Example 1.2

Solution From Fig. 1.3 $VD = u(1-a)D$

$$V = (1-a)u$$

$$\alpha = \frac{\int_{aD}^D u^3 dy}{V^3 D} = \frac{u^3 (D-aD)}{u^3 (1-a)^3 D} = \frac{1}{(1-a)^2}$$

$$\beta = \frac{\int_{aD}^D u^2 dy}{V^2 D} = \frac{u^2 D(1-a)}{u^2 (1-a)^2 D} = \frac{1}{(1-a)}$$

Pressure In some curvilinear flows, the piezometric pressure head may have non-linear variations with depth. The piezometric head h_p at any depth y from the free surface can be expressed as

$$h_p = Z_0 + y + (h_1 - y) + \Delta h$$

$$h_p = Z_0 + h_1 + \Delta h \quad (1.10)$$

in which Z_0 = elevation of the bed, h_1 pressure head at the bed if linear variation of pressure with depth existed and Δh = deviation from the linear pressure head variation at any depth y . For one-dimensional analysis, a representative piezometric head for the section called *effective piezometric head*, h_{ep} , is defined as

$$h_{ep} = Z_0 + h_1 + \frac{1}{h_1} \int_0^{h_1} (\Delta h) dy \quad (1.11)$$

$$= Z_0 + h_1 + \overline{\Delta h} \quad (1.11a)$$

Usually hydrostatic pressure variation is considered as the reference linear variation.

1.6 PRESSURE DISTRIBUTION

The intensity of pressure for a liquid at its free surface is equal to that of the surrounding atmosphere. Since the atmospheric pressure is commonly taken as a reference and of value equal to zero, the free surface of the liquid is thus a surface of zero pressure. The distribution of pressure in an open channel flow is governed by the acceleration due to gravity g and other accelerations and is given by the Euler's equation as below:

In any arbitrary direction s ,

$$-\frac{\partial(p + \gamma Z)}{\partial s} = \rho a_s \quad (1.12)$$

and in the direction normal to s direction, i.e., in the n direction,

$$-\frac{\partial}{\partial n}(p + \gamma Z) = \rho a_n \quad (1.13)$$

in which p = pressure, a_s = acceleration component in the s direction, a_n = acceleration in the n direction and Z = elevation measured above a datum.

Consider the s direction along the streamline and the n direction across it. The direction of the normal towards the centre of curvature is considered as positive. We are interested in studying the pressure distribution in the n -direction. The normal acceleration of any streamline at a section is given by

$$a_n = \frac{v^2}{r} \quad (1.14)$$

where v = velocity of flow along the streamline of radius of curvature r .

Hydrostatic Pressure Distribution The normal acceleration a_n will be zero

- (i) if $v = 0$, i.e., when there is no motion, or
- (ii) if $r \rightarrow \infty$, i.e., when the streamlines are straight lines.

Consider the case of no motion, i.e. the still water case (Fig. 1.4(a)). From Eq. 1.13, since $a_n = 0$, taking n in the Z direction and integrating

$$\frac{p}{\gamma} + Z = \text{constant} = C \quad (1.15)$$

At the free surface [point 1 in Fig. 1.3(a)] $p_1/\gamma = 0$ and $Z = Z_1$, giving $C = Z_1$. At any point A at a depth y below the free surface,

$$\frac{p_A}{\gamma} = (Z_1 - Z_A) = y$$

i.e.

$$p_A = \gamma y \quad (1.16)$$

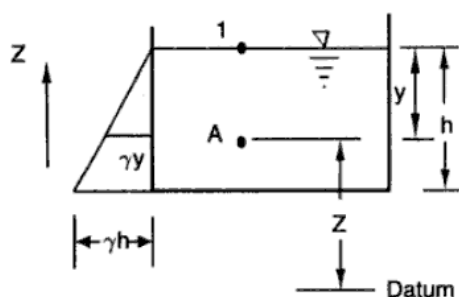


Fig. 1.4(a) Pressure distribution in still water

This linear variation of pressure with depth having the constant of proportionality equal to the unit weight of the liquid is known as *hydrostatic-pressure distribution*.

Channels with Small Slope Let us consider a channel with a very small value of the longitudinal slope θ . Let $\theta \sim \sin \theta \sim 1/1000$. For such channels the vertical section is practically the same

as the normal section. If a flow takes place in this channel with the water surface parallel to the bed, i.e. uniform flow, the streamlines will be straight lines and as such in a vertical direction [Section 0–1 in Fig. 1.4(b)] the normal acceleration $a_n = 0$.

Following the argument of the previous paragraph, the pressure distribution at the Section 0–1 will be hydrostatic. At any point A at a depth y below the water surface,

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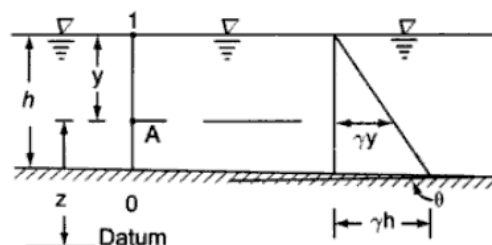


Fig. 1.4(b) Pressure distribution in a channel with small slope

$$\frac{p}{\gamma} = y \quad \text{and} \quad \frac{p}{\gamma} + Z = Z_1$$

= Elevation of water surface

Thus the piezometric head at any point in the channel will be equal to the water-surface elevation. The hydraulic grade line will therefore lie essentially on the water surface.

Channels with Large Slope Figure 1.5 shows a uniform free-surface flow in a channel with a large value of inclination θ . The flow is uniform, i.e. the water surface is parallel to the bed. An element of length ΔL and unit width is considered at the Section 0–1.

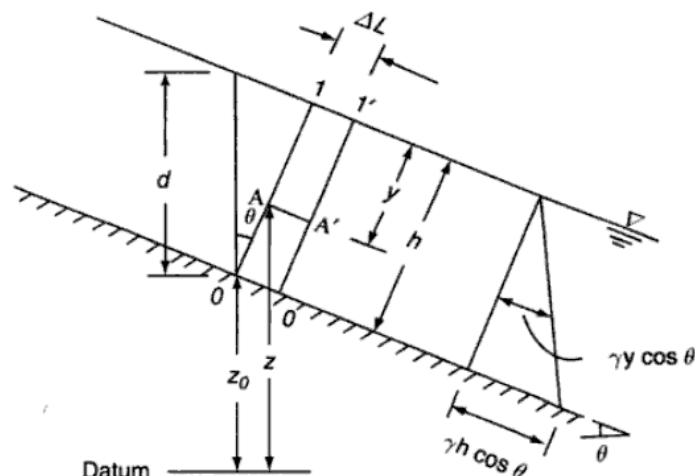


Fig. 1.5 Pressure distribution in a channel with large slope

At any point A at a depth y measured normal to the water surface, the weight of column $A11'A' = \gamma \Delta L y$ and acts vertically downwards. The pressure at AA' supports the normal component of the column $A11'A'$. Thus

$$p_A \Delta L = \gamma y \Delta L \cos \theta \quad (1.17)$$

i.e.
$$p_A = \gamma y \cos \theta \quad (1.18)$$

or
$$p_A / \gamma = y \cos \theta \quad (1.18a)$$

The pressure p_A varies linearly with the depth y but the constant of proportionality is $\gamma \cos \theta$. If h = normal depth of flow, the pressure on the bed at point O , $p_0 = \gamma h \cos \theta$.

If d = vertical depth to water surface measured at the point O , then $h = d \cos \theta$ and the pressure head at point O , on the bed is given by

$$\frac{p_0}{\gamma} = h \cos \theta = d \cos^2 \theta \quad (1.19)$$



The piezometric height at any point $A = Z + y \cos \theta = Z_0 + h \cos \theta$. Thus for channels with large values of the slope, the conventionally defined hydraulic gradient line does not lie on the water surface.

Channels of large slopes are encountered rarely in practice except, typically in spillways and chutes. On the other hand, most of the canals, streams and rivers with which a hydraulic engineer is commonly associated will have slopes ($\sin \theta$) smaller than $1/100$. For such cases $\cos \theta \approx 1.0$. As such, in further sections of this book the term $\cos \theta$ in the expression for the pressure will be omitted with the knowledge that it has to be used as in Eq. 1.18 if θ is large.

1.7 PRESSURE DISTRIBUTION IN CURVILINEAR FLOWS

Figure 1.6(a) shows a curvilinear flow in a vertical plane on an upward convex surface. For simplicity consider a Section 01A2 in which the r direction and Z direction coincide. Replacing the n direction in Eq. 1.13 by $(-r)$ direction,

$$\frac{\partial}{\partial r} \left[\frac{p}{\gamma} + Z \right] = \frac{a_n}{g} \quad (1.20)$$

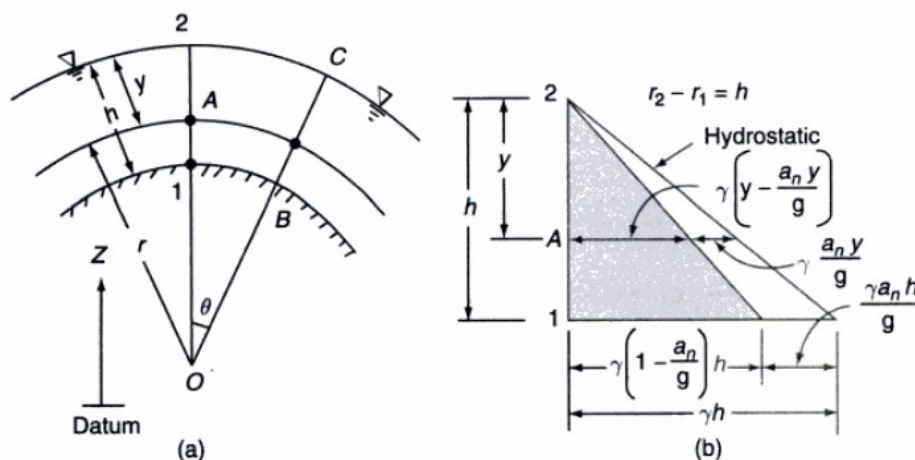


Fig. 1.6 Convex curvilinear flow

Let us assume a simple case in which $a_n = \text{constant}$. Then, the integration of Eq. 1.20 yields

$$\frac{p}{\gamma} + Z = \frac{a_n}{g} r + C \quad (1.21)$$

in which $C = \text{constant}$. With the boundary condition that at point 2 which lies on the free surface, $r = r_2$ and $p/\gamma = 0$ and $Z = Z_2$,

$$\frac{p}{\gamma} = (Z_2 - Z) - \frac{a_n}{g} (r_2 - r) \quad (1.22)$$

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Let $Z_2 - Z$ = depth below the free surface of any point A in the Section $O1A2 = y$. Then for point A ,

$$(r_2 - r) = y = (Z_2 - Z)$$

and

$$\frac{p}{\gamma} = y - \frac{a_n}{g} y \quad (1.23)$$

Equation 1.23 shows that the pressure is less than the pressure obtained by the hydrostatic distribution [Fig. 1.6(b)].

For any normal direction OBC in Fig. 1.6(a), at point C , $(p/\gamma)_c = 0$, $r_c = r_2$, and for any point at a radial distance r from the origin O , by Eq. 1.22

$$\frac{p}{\gamma} = (Z_c - Z) - \frac{a_n}{g} (r_2 - r)$$

But

$$Z_c - Z = (r_2 - r) \cos \theta,$$

giving

$$\frac{p}{\gamma} = (r_2 - r) \cos \theta - \frac{a_n}{g} (r_2 - r) \quad (1.24)$$

It may be noted that when $a_n = 0$, Eq. 1.24 is the same as Eq. 1.18a, for the flow down a steep slope.

If the curvature is convex downwards, (i.e. r direction is opposite to Z direction) following the argument as above, for constant a_n , the pressure at any point A at a depth y below the free surface in a vertical Section $O1A2$ [Fig. 1.7(a)] can be shown to be

$$\frac{p}{\gamma} = y + \frac{a_n}{g} y \quad (1.25)$$

The pressure distribution in a vertical section is as shown in Fig. 1.7(b).

Thus it is seen that for a curvilinear flow in a vertical plane, an additional pressure will be imposed on the hydrostatic pressure distribution. The extra pressure will be additive if the curvature is convex downwards and subtractive if it is convex upwards.

Normal Acceleration In the previous discussion on curvilinear flows, the normal acceleration a_n was assumed to be constant. However, it is known that at any point in a curvilinear flow, $a_n = \frac{v^2}{r}$, where v = velocity and r = radius of curvature of the streamline at that point.

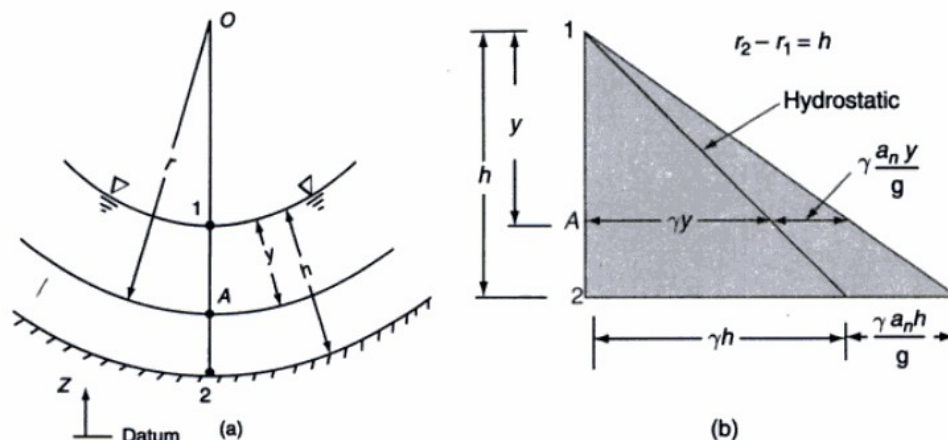


Fig. 1.7 Concave curvilinear flow

In general, one can write $v = f(r)$ and the pressure distribution can then be expressed by

$$\left(\frac{p}{\gamma} + Z \right) = \int \frac{v^2}{gr} dr + \text{Const} \quad (1.26)$$

This expression can be evaluated if $v = f(r)$ is known. For simple analysis, the following functional forms are used in appropriate circumstances:

- (i) $v = \text{constant} = V = \text{mean velocity of flow}$
- (ii) $v = c/r$, (free-vortex model)
- (iii) $v = cr$, (forced-vortex model)
- (iv) $a_n = \text{constant} = V^2/R$, where $R = \text{radius of curvature at mid-depth}$.

Example 1.3 At a section in a rectangular channel, the pressure distribution was recorded as shown in Fig. 1.8. Determine the effective piezometric head for this section. Take the hydrostatic pressure distribution as the reference.

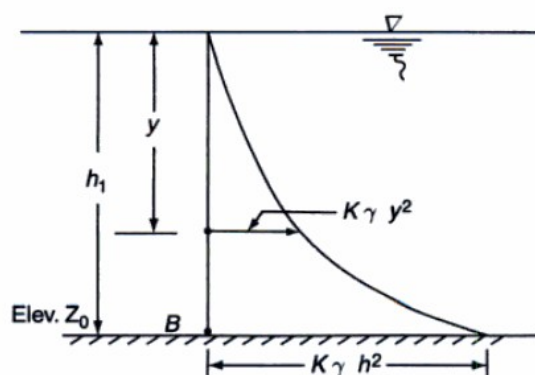


Fig.1.8 Example 1.3

Solution Z_0 = elevation of the bed of channel above the datum
 h_1 = depth of flow at the section
 OB

Let h_p = piezometric head at point A, depth y below the free surface

Then $h_p = Z_0 + ky^2 + (h_1 - y)$

Putting $h_p = Z_0 + h_1 + \Delta h$

$\Delta h = ky^2 - y$

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Effective piezometric head, by Eq. 1.11 is

$$\begin{aligned} h_{ep} &= Z_0 + h_1 + \frac{1}{h_1} \int_0^{h_1} (\Delta h) dy \\ &= Z_0 + h_1 + \frac{1}{h_1} \int_0^{h_1} (ky^2 - y) dy \\ h_{ep} &= Z_0 + \frac{h_1}{2} + \frac{kh_1^2}{2} \end{aligned}$$

Example 1.4 A spillway bucket has a radius of curvature R as shown in Fig. 1.9. (a) Obtain an expression for the pressure distribution at a radial section of inclination θ to the vertical. Assume the velocity at any radial section to be uniform and the depth of flow h to be constant. (b) What is the effective piezometric head for the above pressure distribution?

Solution (a) Consider the Section 012. Velocity = V = constant across 12. Depth of flow = h . From Eq. 1.26, since the curvature is convex downwards

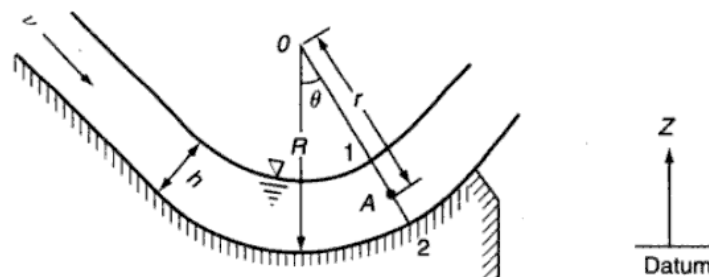


Fig. 1.9 Example 1.4

$$\begin{aligned} \left(\frac{p}{\gamma} + Z \right) &= \int \frac{v^2}{gr} dr + \text{Const} \\ \frac{p}{\gamma} + Z &= \frac{V^2}{g} \ln r + C \end{aligned} \quad (1.27)$$

At the point 1, $p/\gamma = 0$, $Z = Z_1$, $r = R - h$

$$C = Z_1 - \frac{V^2}{g} \ln(R - h)$$

At any point A, at radial distance r from O

$$\frac{p}{\gamma} = (Z_1 - Z) \frac{V^2}{g} \ln \left(\frac{r}{R - h} \right) \quad (1.28)$$

But $(Z_1 - Z) = (r - R + h) \cos \theta$



$$\frac{p}{\gamma} = (r - R + h) \cos \theta + \frac{V^2}{g} \ln \left(\frac{r}{R - h} \right) \quad (1.29)$$

Equation 1.29 represents the pressure distribution at any point (r, θ) . At point 2, $r = R$, $p = p_2$.

(b) Effective piezometric head, h_{ep} :

From Eq. 1.28 the piezometric head h_p at A is

$$h_p = \left(\frac{p}{\gamma} + Z \right)_A = Z_1 + \frac{V^2}{g} \ln \left(\frac{r}{R - h} \right)$$

Noting that $Z_1 = Z_2 + h \cos \theta$ and expressing h_p in the form of Eq. 1.10

$$h_p = Z_2 + h \cos \theta + \Delta h$$

Where

$$\Delta h = \frac{V^2}{g} \ln \frac{r}{R - h}$$

The effective piezometric head h_{ep} from Eq. 1.11 is

$$h_{ep} = Z_2 + h \cos \theta + \frac{1}{h \cos \theta} \int_{R-h}^R \frac{V^2}{g} \ln \frac{r}{R - h} dr$$

on integration,

$$\begin{aligned} h_{ep} &= Z_2 + h \cos \theta + \frac{V^2}{gh \cos \theta} \left[-h + R \ln \frac{R}{R - h} \right] \\ &= Z_2 + h \cos \theta + \frac{V^2 \left(-\frac{h}{R} + \ln \frac{1}{1 - h/R} \right)}{gR \left(\frac{h}{R} \right) \cos \theta} \end{aligned} \quad (1.30)$$

It may be noted that when $R \rightarrow \infty$ and $h/R \rightarrow 0$, $h_{ep} \rightarrow Z_2 + h \cos \theta$

1.8 FLOWS WITH SMALL WATER-SURFACE CURVATURE

Consider a free-surface flow with a convex upward water surface over a horizontal bed (Fig. 1.10). For this water surface, d^2h/dx^2 is negative. The radius of curvature of the free surface is given by

$$\frac{1}{r_1} = \frac{\frac{d^2h}{dx^2}}{\left[1 + \left(\frac{dh}{dx} \right)^2 \right]^{3/2}} \approx \frac{d^2h}{dx^2} \quad (1.31)$$

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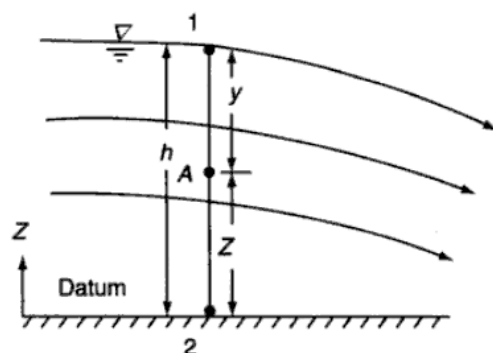


Fig. 1.10 Definition sketch of flow with small water-surface curvature

Assuming linear variation of the curvature with depth, at any point A at a depth y below the free surface, the radius of curvature r is given by

$$\frac{1}{r} = \frac{d^2h}{dx^2} \left(\frac{h-y}{h} \right) \quad (1.32)$$

If the velocity at any depth is assumed to be constant and equal to the mean velocity V in the section, the normal acceleration a_n at point A is given by

$$a_n = \frac{V^2}{r} = \frac{V^2(h-y)}{h} \frac{d^2h}{dx^2} = K(h-y) \quad (1.33)$$

where $K = (V^2/h)d^2h/dx^2$. Taking the channel bed as the datum, the piezometric head h_p at point A is then by Eq. 1.26

$$h_p = \left(\frac{p}{\gamma} + Z \right) = \int \frac{K}{g} (h-y) dy + \text{Const}$$

i.e.

$$h_p = \frac{K}{g} \left(hy - \frac{y^2}{2} \right) + C \quad (1.34)$$

Using the boundary condition; at $y = 0$, $p/\gamma = 0$, $Z = h$ and $h_p = h$, leads to $C = h$,

$$h_p = h + \frac{K}{g} \left(hy - \frac{y^2}{2} \right) \quad (1.35)$$

Equation 1.35 gives the variation of the piezometric head with the depth y below the free surface. Designing $h_p = h + \Delta h$

$$\Delta h = \frac{K}{g} \left(hy - \frac{y^2}{2} \right)$$

The mean value of Δh

$$\begin{aligned} \overline{\Delta h} &= \frac{1}{h} \int_0^h \Delta h dy \\ &= \frac{K}{gh} \int_0^h \left(hy - \frac{y^2}{2} \right) dy = \frac{Kh^2}{3g} \end{aligned}$$

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The *effective piezometric head* h_{ep} at the section with the channel bed as the datum can now be expressed as

$$h_{ep} = h + \frac{Kh^2}{3g} \quad (1.36)$$

It may be noted that d^2h/dx^2 and hence K is negative for convex upward curvature and positive for concave upward curvature. Substituting for K , Eq. 1.36 reads as

$$h_{ep} = h + \frac{1}{3} \frac{V^2 h}{g} \frac{d^2 h}{dx^2} \quad (1.37)$$

This equation, attributed to Boussinesq⁴ finds application in solving problems with small departures from the hydrostatic pressure distribution due to the curvature of the water surface.

1.9 EQUATION OF CONTINUITY

The continuity equation is a statement of the law of conservation of matter. In open-channel flows, since we deal with incompressible fluids, this equation is relatively simple and much more for the cases of steady flow.

Steady Flow In a steady flow the volumetric rate of flow (discharge in m³/s) past various section must be the same. Thus in a varied flow, if Q = discharge, V = mean velocity and A = area of cross-section with suffixes representing the sections to which they refer

$$Q = VA = V_1 A_1 = V_2 A_2 = \dots \quad (1.38)$$

If the velocity distribution is given, the discharge is obtained by integration as in Eq. 1.4. It should be kept in mind that the area element and the velocity through this area element must be perpendicular to each other.

In a steady spatially-varied flow, the discharge at various sections will not be the same. A budgeting of inflows and outflows of a reach is necessary. Consider, for example, an SVF with increasing discharge as in Fig. 1.11. The rate of addition of discharge = $dQ/dx = q_*$. The discharge at any section at a distance x from Section 1

$$= Q = Q_1 + \int_0^x q_* dx \quad (1.39)$$

If q_* = constant, $Q = Q_1 + q_* x$ and $Q_2 = Q_1 + q_* L$

Unsteady Flow In the unsteady flow of incompressible fluids, if we consider a reach of the channel, the continuity equation states that the net discharge going out of all the boundary surfaces of the reach is equal to the rate of depletion of the storage within it.

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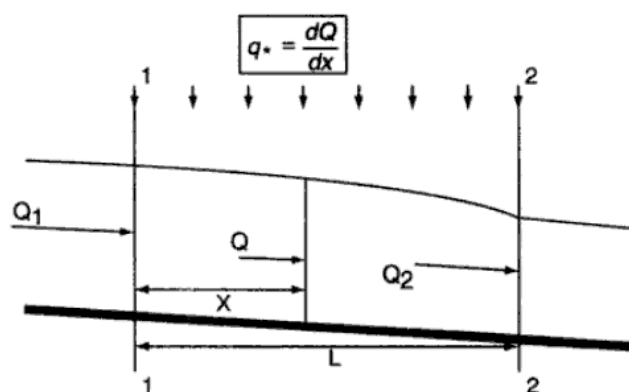


Fig. 1.11 Spatially varied flow

In Fig. 1.12, if $Q_2 > Q_1$, more flow goes out than what is coming into Section 1. The excess volume of outflow in a time Δt is made good by the depletion of storage within the reach bounded by Sections 1 and 2. As a result of this the water surface will start falling. If $\Delta t =$ distance between Sections 1 and 2,

$$Q_2 - Q_1 = \frac{\partial Q}{\partial x} \Delta x$$

The excess volume rate of flow in a time $\Delta t = (\partial Q / \partial x) \Delta x \Delta t$. If the top width of the canal at any depth y is T , $\partial A / \partial y = T$. The storage volume at depth $y = A \Delta x$. The rate of decrease of storage $= -\Delta x \frac{\partial A}{\partial y} \frac{\partial y}{\partial t} = -T \Delta x \frac{\partial y}{\partial t}$. The decrease in storage in time

$$\Delta t = \left(-T \Delta x \frac{\partial y}{\partial t} \right) \Delta t. \text{ By continuity } \frac{\partial Q}{\partial x} \Delta x \Delta t = -T \frac{\partial y}{\partial t} \Delta x \Delta t.$$

$$\text{or } \frac{\partial Q}{\partial x} + T \frac{\partial y}{\partial t} = 0 \quad (1.40)$$

Equation 1.40 is the basic equation of continuity for unsteady, open-channel flow.

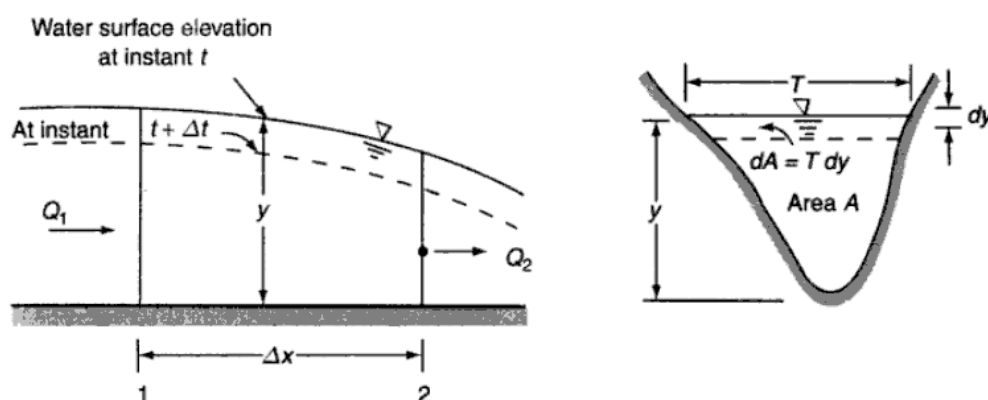


Fig. 1.12 Definition sketch of unsteady flow

Example 1.5

The velocity distribution in the plane of a vertical sluice gate discharging free is shown in Fig. 1.13. Calculate the discharge per unit width of the gate.



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Location	1	2	3	4	5	6	7
Velocity (m/s)	2.3	2.5	2.6	2.6	2.5	2.1	0.0
θ (degrees)	5	10	15	20	25	30	—
y (m)	0.05	0.10	0.15	0.20	0.25	0.30	0.35

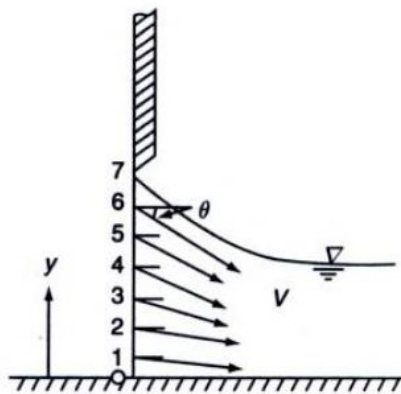


Fig. 1.13 Example 1.5

Solution The component of velocity normal to the y -axis is calculated as $V_n = V \cos \theta$. The discharge per unit width $q = \sum V_n \Delta y$. The velocity is zero at the boundaries, i.e., at end sections 0 and 7 and this should be noted in calculating average velocities relating to the end sections. The calculations are done in tabular form as shown below. In the table $\Delta q =$ discharge in the element between two sections $= \text{Col 5} \times \text{Col 6}$. The total discharge per unit width is $0.690 \text{ m}^3/\text{s}/\text{m}$.

1	2	3	4	5	6	7
Section	$V = \text{Velocity}$ (m/s)	θ°	$V_n = V \cos \theta$ (m/s)	Average V_n (m/s)	Δy (m)	Δq_3 ($\text{m}^3/\text{s}/\text{m}$)
0	0	0	0.000		0	0
1	2.3	5	2.291	1.146	0.05	0.057
2	2.5	10	2.462	2.377	0.05	0.119
3	2.6	15	2.511	2.487	0.05	0.124
4	2.6	20	2.443	2.477	0.05	0.124
5	2.5	25	2.266	2.354	0.05	0.118
6	2.1	30	1.819	2.042	0.05	0.102
7	0	0	0.000	0.090	0.05	0.045
Total Discharge $= q =$						0.690

Example 1.6 While measuring the discharge in a small stream it was found that the depth of flow increases at the ratio of 0.10 m/h. If the discharge at that section was $25 \text{ m}^3/\text{s}$ and the surface width of the stream was 20m, estimate the discharge at a section 1 km upstream.

Solution This is a case of unsteady flow and the continuity equation Eq. 1.40 will be used.

$$T \frac{\partial y}{\partial t} = \frac{20 \times 0.10}{60 \times 60} = 0.000556$$

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By Eq. 1.36,

$$\begin{aligned}\frac{Q_2 - Q_1}{\Delta x} &= \frac{\partial Q}{\partial x} = -T \frac{\partial y}{\partial t} \\ Q_1 &= \text{discharge at the upstream section} \\ &= Q_2 + T \frac{\partial y}{\partial t} \Delta x = 25.0 + 1000 \times 0.000556 \\ &= 25.556 \text{ m}^3/\text{s}\end{aligned}$$

1.10 ENERGY EQUATION

In the one-dimensional analysis of steady open-channel flow, the energy equation in the form of the Bernoulli equation is used. According to this equation, the total energy at a downstream section differs from the total energy at the upstream section by an amount equal to the loss of energy between the sections.

Figure 1.14 shows a steady varied flow in a channel. If the effect of the curvature on the pressure distribution is neglected, the total energy head (in N.m/newton of fluid) at any point *A* at a depth *d* below the water surface is

$$H = Z_A + d \cos \theta + \alpha \frac{V^2}{2g} \quad (1.41)$$

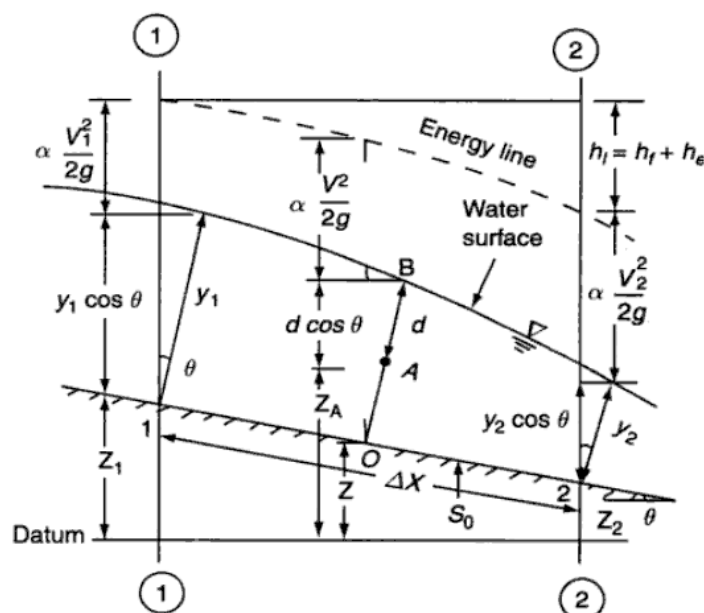


Fig.1.14 Definition sketch for the energy equation

This total energy will be constant for all values of *d* from zero to *y* at a normal section through point *A* (i.e. Section *OAB*), where *y* = depth of flow measured normal to the bed. Thus the total energy at any section whose bed is at an elevation *Z* above the datum is

$$H = Z + y \cos \theta + \alpha V^2 / 2g \quad (1.42)$$



In Fig. 1.14, the total energy at a point on the bed is plotted along the vertical through that point. Thus the elevation of energy line on the line 1–1 represents the total energy at any point on the normal section through point 1. The total energies at normal sections through 1 and 2 are therefore

$$H_1 = Z_1 + y_1 \cos \theta + \alpha_1 \frac{V_1^2}{2g}$$

$$H_2 = Z_2 + y_2 \cos \theta + \alpha_2 \frac{V_2^2}{2g}$$

respectively. The term $(Z + y \cos \theta) = h$ represents the elevation of the hydraulic grade line above the datum.

If the slope of the channel θ is small, $\cos \theta \approx 1.0$, the normal section is practically the same as the vertical section and the total energy at any section can be written as

$$H = Z + y + \alpha \frac{V^2}{2g} \quad (1.43)$$

Since most of the channels in practice happen to have small values of θ ($\theta < 10^\circ$), the term $\cos \theta$ is usually neglected. Thus the energy equation is written as Eq. 1.40 in subsequent sections of this book, with the realisation that the slope term will be included if $\cos \theta$ is appreciably different from unity.

Due to energy losses between Sections 1 and 2, the energy head H_1 will be larger than H_2 and $H_1 - H_2 = h_L$ = head loss. Normally, the head loss (h_L) can be considered to be made up of frictional losses (h_f) and eddy or form loss (h_e) such that $h_L = h_f + h_e$. For prismatic channels, $h_e = 0$. One can observe that for channels of small slope the piezometric head line essentially coincides with the free surface. The energy line which is a plot of H vs x is a dropping line in the longitudinal (x) direction. The difference of the ordinates between the energy line and free surface represents the velocity head at that section. In general, the bottom profile, water-surface and energy line will have distinct slopes at a given section. The bed slope is a geometric parameter of the channel. The slope of the energy line depends on the resistance characteristics of the channel and is discussed in Chapter 3. Discussions on the water-surface profiles are presented in chapter 4 and 5.

In designating the total energy by Eq. 1.41 or 1.42, hydrostatic pressure distribution was assumed. However, if the curvature effects in a vertical plane are appreciable, the pressure distribution at a section may have a non-linear variation with the depth d . In such cases the effective piezometric head h_{ep} as defined in Eq. 1.11 will be used to represent the total energy at a section as

$$H = h_{ep} + \alpha \frac{V^2}{2g} \quad (1.44)$$

Example 1.7

The width of a horizontal rectangular channel is reduced from 3.5 m to 2.5 m and the floor is raised by 0.25 m in elevation at a given section. At the upstream section, the depth of flow is 2.0 m and the kinetic energy correction

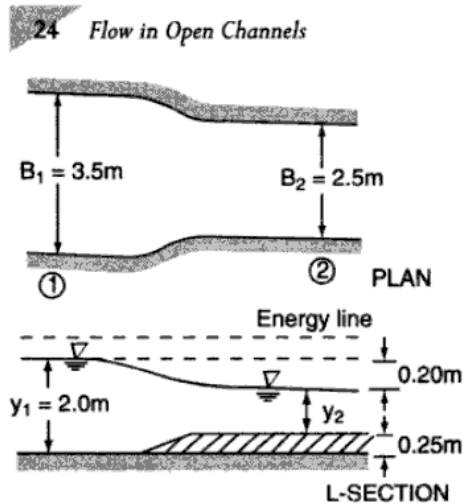


Fig. 1.15 Example 1.7

factor α is 1.15. If the drop in the water surface elevation at the contraction is 0.20 m, calculate the discharge if (a) the energy loss is neglected, and (b) the energy loss is one-tenth of the upstream velocity head. [The kinetic energy correction factor at the contracted section may be assumed to be unity].

Solution Referring to Fig. 1.15,
 $y_1 = 2.0$ m

$$y_2 = 2.0 - 0.25 - 0.20 = 1.55 \text{ m}$$

By continuity

$$B_1 y_1 V_1 = B_2 y_2 V_2$$

$$V_1 = \frac{2.5 \times 1.55}{3.5 \times 2.0} V_2 = 0.5536 V_2$$

(a) When there is no energy loss

By energy equation applied to Sections 1 and 2,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g}$$

$$\alpha_1 = 1.15 \text{ and } \alpha_2 = 1.0$$

$$\frac{V_2^2 - (1.15 V_1^2)}{2g} = y_1 - y_2 - \Delta Z$$

$$\frac{V_2^2}{2g} [1 - (1.15)(0.5536)^2] = 2.00 - 1.55 - 0.25$$

$$0.6476 \frac{V_2^2}{2 \times 9.81} = 0.2$$

$$V_2 = 2.462 \text{ m/s}$$

$$\text{Discharge } Q = 2.5 \times 1.55 \times 2.462 = 9.54 \text{ m}^3/\text{s}$$

(b) When there is an energy loss

$$H_L = 0.1 \left[\alpha_1 \frac{V_1^2}{2g} \right] = 0.115 \frac{V_1^2}{2g}$$

By energy equation,

$$Z_1 + y_1 + \alpha_1 \frac{V_1^2}{2g} = (Z_1 + \Delta Z) + y_2 + \alpha_2 \frac{V_2^2}{2g} + H_L$$

$$\left[\alpha_2 \frac{V_2^2}{2g} - \alpha_1 \frac{V_1^2}{2g} + H_L \right] = y_1 - y_2 - \Delta Z$$

Substituting $\alpha_2 = 1.0$, $\alpha_1 = 1.15$ and $H_L = 0.115 \frac{V_1^2}{2g}$

$$\frac{V_2^2}{2g} - 1.15 \frac{V_1^2}{2g} - 0.115 \frac{V_1^2}{2g} = 2.00 - 1.55 - 0.25$$

Since $V_1 = 0.5536 V_2$

$$\frac{V_2^2}{2g} \left[1 - (0.9)(1.15)(0.5536)^2 \right] = 0.2$$

$$\frac{0.6826 V_2^2}{2 \times 9.81} = 0.2$$

$V_2 = 2.397$ m/s and discharge $Q = 2.5 \times 1.55 \times 2.397 = 9.289$ m³/s

Example 1.8 A sluice gate in a 2.0-m wide horizontal rectangular channel is discharging freely as shown in Fig. 1.16. If the depths a small distance upstream (y_1) and downstream (y_2) are 2.5 m and 0.20 m respectively, estimate the discharge in the channel (i) by neglecting energy losses at the gate, and (ii) by assuming the energy loss at the gate to be 10% of the upstream depth y_1 .

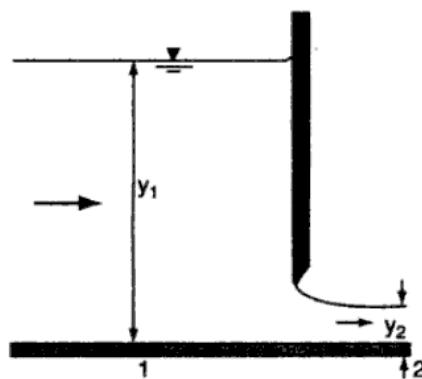


Fig. 1.16 Free flow from a sluice gate -- Example 1.8

Solution Referring to Fig. 1.16, $y_1 = 2.5$ m and $y_2 = 0.20$ m

$$B y_1 V_1 = B y_2 V_2$$

$$V_2 = \frac{y_1}{y_2} V_1 = \frac{2.5}{0.20} \times V_1 = 12.5 V_1$$

(i) When there is no energy loss

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g}$$

Since the channel is horizontal, $Z_1 = Z_2$ and

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = (y_1 - y_2)$$

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$$\frac{V_1^2}{2g}[(12.5)^2 - 1] = 2.50 - 0.20 = 2.30$$

$$\frac{V_1^2}{2g} = \frac{2.30}{155.25} = 0.01481 \text{ and } V_1 = 0.539 \text{ m/s.}$$

$$\text{Discharge } Q = By_1V_1 = 2.0 \times 2.5 \times 0.539 = 2.696 \text{ m}^3/\text{s.}$$

(ii) *When there is energy loss*

$$H_L = \text{Energy loss} = 0.10 y_1 = 0.25 \text{ m}$$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + H_L$$

$$\frac{V_2^2}{2g} - \frac{V_1^2}{2g} = (y_1 - y_2 - H_L)$$

$$\frac{V_1^2}{2g}[(12.5)^2 - 1] = 2.50 - 0.20 - 0.25 = 2.05$$

$$\frac{V_1^2}{2g} = \frac{2.05}{155.25} = 0.0132 \text{ and } V_1 = 0.509 \text{ m/s}$$

$$\text{Discharge } Q = By_1V_1 = 2.0 \times 2.5 \times 0.509 = 2.545 \text{ m}^3/\text{s.}$$

1.11 MOMENTUM EQUATION

Steady Flow Momentum is a vector quantity. The momentum equation commonly used in most of the open channel flow problems is the *linear-momentum equation*. This equation states that the algebraic sum of all external forces, acting in a given direction on a fluid mass equals the time rate of change of linear-momentum of the fluid mass in the direction. In a steady flow the rate of change of momentum in a given direction will be equal to the net flux of momentum in that direction.

Figure 1.17 shows a *control volume* (a volume fixed in space) bounded by Sections 1 and 2, the boundary and a surface lying above the free surface. The various forces acting on the control volume in the longitudinal direction are as follows:

- (i) Pressure forces acting on the control surfaces, F_1 and F_2 .
- (ii) Tangential force on the bed, F_3 ,
- (iii) Body force, i.e., the component of the weight of the fluid in the longitudinal direction, F_4 .

By the linear-momentum equation in the longitudinal direction for a steady-flow discharge of Q ,

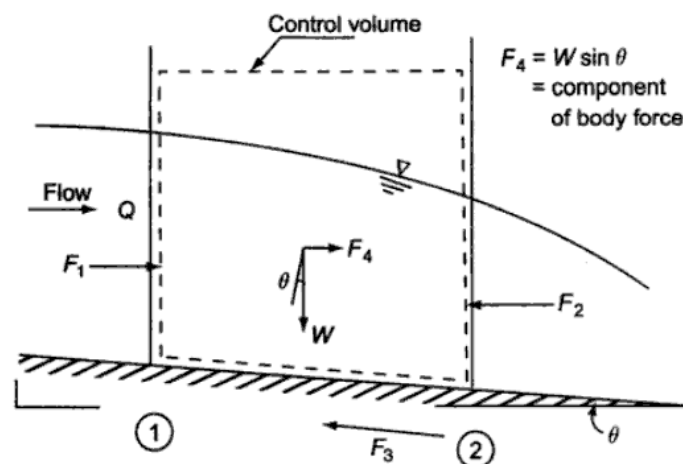


Fig.1.17 Definition sketch for the momentum equation

$$\Sigma F_1 = F_1 - F_2 - F_3 + F_4 = M_2 - M_1 \quad (1.45)$$

in which $M_1 = \beta_1 \rho Q V_1$ = momentum flux entering the control volume, $M_2 = \beta_2 \rho Q V_2$ = momentum flux leaving the control volume.

In practical applications of the momentum equation, the proper identification of the geometry of the control volume and the various forces acting on it are very important. The momentum equation is a particularly useful tool in analysing rapidly varied flow (RVF) situations where energy losses are complex and cannot be easily estimated. It is also very helpful in estimating forces on a fluid mass. Detailed information on the basis of the momentum equation and selection of the control volume are available in books dealing with the mechanics of fluids.^{5,6}

Example 1.9

Estimate the force, on a sluice gate shown in Fig. 1.18.

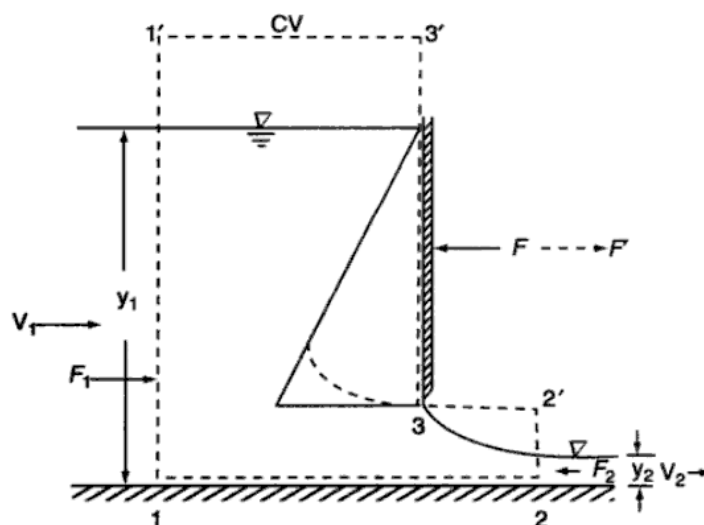


Fig. 1.18 Forces in a sluice gate flow-Example 1.9

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Solution Consider a unit width of the channel. The force exerted on the fluid by the gate is F , as shown in the figure. This is equal and opposite to the force exerted by the fluid on the gate, F' .

Consider the control volume as shown by dotted lines in the figure. Section 1 is sufficiently far away from the efflux section and hydrostatic pressure distribution can be assumed. The frictional force on the bed between Sections 1 and 2 is neglected. Also assumed are $\beta_1 = \beta_2 = 1.0$. Section 2 is at the vena contracta of the jet where the streamlines are parallel to the bed. The forces acting on the control volume in the longitudinal direction are

$$F_1 = \text{pressure force on the control surface at Section 1} = \frac{1}{2} \gamma y_1^2$$

$$F_2 = \text{pressure force on the control surface at Section 2} = \frac{1}{2} \gamma y_2^2 \quad \text{acting in a direction opposing } F_1.$$

$$F = \text{reaction force of the gate on the Section 3}$$

By the momentum equation, Eq. 1.45,

$$\frac{1}{2} \gamma y_1^2 - \frac{1}{2} \gamma y_2^2 - F = \rho q (V_2 - V_1) \quad (1.46)$$

in which $q = \text{discharge per unit width} = V_1 y_1 = V_2 y_2$. Simplifying Eq. 1.46,

$$F = \frac{1}{2} \gamma \frac{(y_1 - y_2)}{y_1 y_2} \left(y_1 y_2 (y_1 + y_2) - \frac{2q^2}{g} \right) \quad (1.47)$$

If the loss of energy between Sections 1 and 2 is assumed to be negligible, by the energy equation with $\alpha_1 = \alpha_2 = 1.0$

$$y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} \quad (1.48)$$

Substituting

$$V_1 = \frac{q}{y_1} \quad \text{and} \quad V_2 = \frac{q}{y_2}$$

$$\frac{q^2}{g} = \frac{2y_1^2 y_2^2}{(y_1 + y_2)}$$

and by Eq. 1.43,

$$F = \frac{1}{2} \gamma \frac{(y_1 - y_2)^3}{(y_1 + y_2)} \quad (1.49)$$

The force on the gate F' would be equal and opposite to F .

Example 1.10 Figure 1.19 shows a hydraulic jump in a horizontal apron aided by a two dimensional block on the apron. Obtain an expression for the drag force per unit length of the block.

Solution Consider a control volume surrounding the block as shown in Fig. 1.19. A unit width of apron is considered. The drag force on the block would have a reaction force = F_D on the control surface, acting in the upstream direction as shown in Fig. 1.19. Assume, a frictionless, horizontal channel and hydrostatic pressure distribution at Sections 1 and 2.

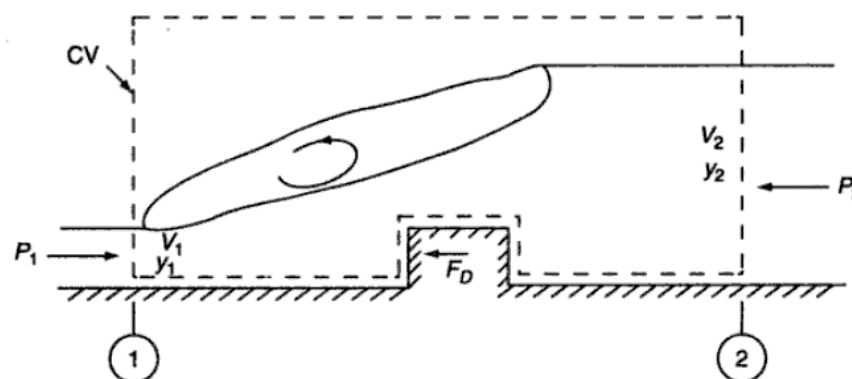


Fig. 1.19 Example 1.10

By momentum equation, Eq. 1.45, in the direction of the flow

$$P_1 - F_D - P_2 = M_2 - M_1$$

$$\frac{1}{2} \gamma y_1^2 - F_D - \frac{1}{2} \gamma y_2^2 = \rho q (\beta_2 V_2 - \beta_1 V_1)$$

where q = discharge per unit width of apron = $y_1 V_1 = y_2 V_2$.

Assuming $\beta_1 = \beta_2 = 1.0$

$$\begin{aligned} F_D &= \frac{1}{2} \gamma y_1^2 - \frac{1}{2} \gamma y_2^2 = \rho q^2 \left(\frac{1}{y_2} - \frac{1}{y_1} \right) \\ &= \frac{\gamma}{2} \left[y_1^2 - y_2^2 + \frac{q^2}{g} \left(\frac{y_2 - y_1}{y_1 y_2} \right) \right] \end{aligned}$$

Unsteady Flow In unsteady flow, the linear-momentum equation will have an additional term over and above that of the steady flow equation to include the rate of change of momentum in the control volume. The momentum equation would then state that in an unsteady flow the algebraic sum of all external forces in a given direction on a fluid mass equals the net change of the linear-momentum flux of the fluid mass in that direction plus the time rate of increase of momentum in that direction within the control volume. An application of the momentum equation in unsteady flows is given in Chapter 10. For details on the momentum equation in unsteady flow consult References 5 and 6.

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Specific Force The steady-state momentum equation (Eq. 1.45) takes a simple form if the tangential force F_3 and body force F_4 are both zero. In that case

$$F_1 - F_2 = M_2 - M_1$$

or

$$F_1 + M_1 = F_2 + M_2$$

Denoting $\frac{1}{\gamma}(F + M) = P_s$

$$(P_s)_1 = (P_s)_2 \quad (1.50)$$

The term P_s is known as the *specific force* and represents the sum of the pressure force and momentum flux per unit weight of the fluid at a section. Equation (1.50) states that the specific force is constant in a horizontal, frictionless channel. This fact can be advantageously used to solve some flow situations. An application of the specific force relationship to obtain an expression for the depth at the end of a hydraulic jump is given in Section 6.4. In a majority of applications the force F is taken as due to hydrostatic pressure distribution and hence is given by, $F = \gamma A \bar{y}$ where $\bar{y}$ is the depth of the centre of gravity of the flow area.

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**PROBLEMS***Problem Distribution*

Topic	Problems
Classification	1.1
α and β	1.2 – 1.6
Pressure distribution	1.7 – 1.14
Continuity equation	1.15 – 1.17
Energy equation	1.18 – 1.22
Momentum equation	1.23 – 1.31

1.1 Classify the following open-channel flow situations:

- (a) Flow from a sluice gate
- (b) Flow in a main irrigation canal
- (c) A river during flood

- (d) Breaking of a dam
 (e) Flow over a spillway
 (f) Sudden opening of a sluice gate
 (g) Spreading of irrigation water on a field
 (h) Flow in a sewer
- 1.2 The Velocity distributions along the vertical in an open channel are as shown in Fig. 1.20. Determine the kinetic energy correction factor α and momentum correction factor β for both the velocity profiles.

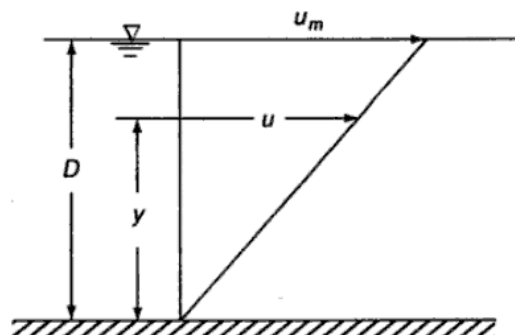


Fig. 1.20 Problem 1.2

- 1.3 The velocity distribution along a vertical in a channel can be expressed as $v/v_{\max} = (y/y_0)^{1/2}$ where y_0 = depth of flow, v = velocity at any height y above the bed and n = a constant. Find the values of α and β .
- 1.4 For the velocity distribution given in Fig. 1.21, find α and β .

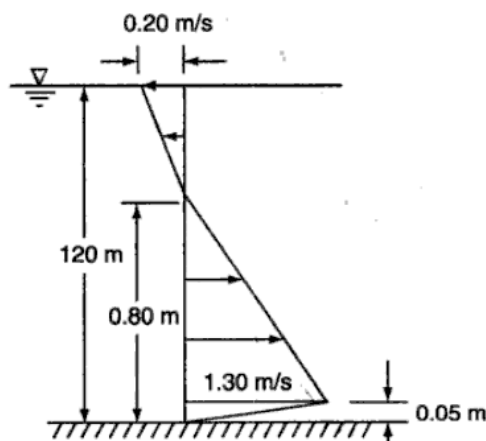


Fig. 1.21 Problem 1.4

- 1.5 The velocity distribution in a channel is given by $u = u(y)$. By representing $u = V + \delta u$, where V = mean velocity and δu = deviation from the mean, show that

$$\alpha \approx 1 + 3\eta \quad \text{and} \quad \beta \approx 1 + \eta$$

where

$$\eta = \frac{1}{AV^2} \int_A (\delta u)^2 dA$$

- 1.6 A rectangular channel curved in the vertical plane is 2.0 m wide and has a centreline radius of 5.0 m. The velocity distribution at a radial section can be considered to be an irrotational vortex, i.e. $v = C/r$. The depth of flow is 1.50 m and is constant along the channel.

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For a discharge of $6.0 \text{ m}^3/\text{s}$, find (a) the velocity distribution, (b) the average velocity, and (c) the correction factors α and β .

- 1.7 For the following two pressure distributions [Fig 1.22 (a) and (b)] in an open channel flow, calculate the effective piezometric head. Take the hydrostatic pressure distribution as the reference.

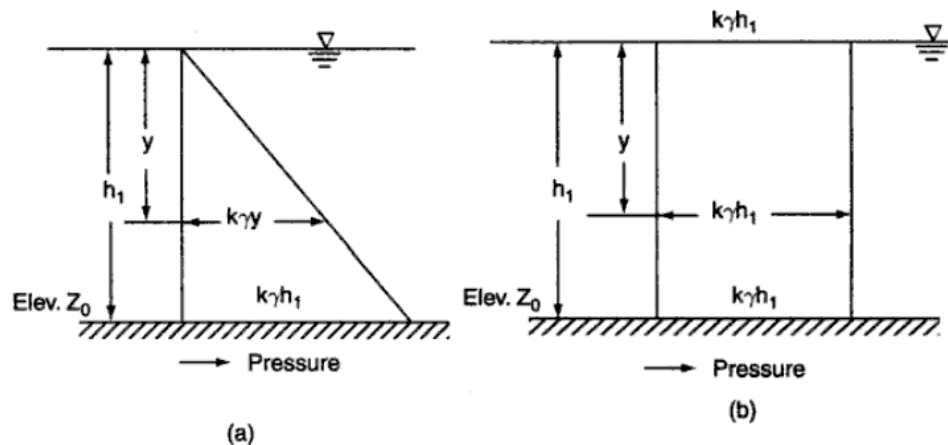


Fig. 1.22 Problem 1.7

- 1.8 A rectangular channel has a convex curvature in a vertical plane on its bed. At a section the bed has an inclination of 30° to the horizontal and the depth measured normal to the flow is 0.75 m . A certain flow produces a normal acceleration of $0.4 g$ which can be assumed to be constant throughout the depth. Determine the pressure distribution and compare it with the hydrostatic distribution.
- 1.9 For the situation detailed in Problem 1.8, determine the pressure distribution if the boundary has a concave curvature to the flow and the rest of the data remain same.
- 1.10 In a flow over a certain spillway crest the normal acceleration a_n can be assumed to be constant. Show that the pressure on the crest is atmospheric when $a_n = g \cos \theta$, where θ = inclination of the normal to the surface with the vertical.
- 1.11 Assuming the flow in the spillway bucket of Example 1.3 to be an irrotational vortex ($v = C/r$) at a constant depth h in the curved portion, show that

$$\frac{p_2}{\gamma} = h \cos \theta + \frac{v_1^2 - v_2^2}{2g}$$

- 1.12 If the flow Problem 1.11 is assumed to be a forced vortex ($v = Cr$), show that

$$\frac{p_2}{\gamma} = h \cos \theta + \frac{v_2^2 - v_1^2}{2g}$$

- 1.13 A spillway crest having a circular arc of radius 6.0 m is shown in Fig. 1.23. Estimate the pressure at point 1 when the discharge intensity is $5.0 \text{ m}^3/\text{s}$ per metre width by assuming:
- Velocity is constant across 1-2
 - Velocity varies linearly with the radius ($v = Cr$)
 - Velocity is inversely proportional to the radius ($v = C/r$)
 - Normal acceleration is constant at a value corresponding to average values of velocity and radius

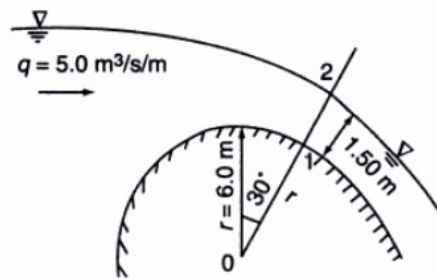


Fig. 1.23 Problem 1.13

- 1.14 At a free overflow in a wide horizontal rectangular channel the depth at the brink was found to be 0.86 m. At a section that is 5 m upstream of the brink, the depth was noted as 1.20 m. The discharge per unit width of the channel is estimated as 4.12 m³/s per m width. Assuming the water surface profile between the above two sections to be given by $y = Ax^2 + B$, where x is measured from the upstream section, determine the
- pressure distribution at a 2.5 m upstream section of the brink, and
 - effective piezometric head at (i) $x = 1.0$ m, and (ii) $x = 2.5$ m.
- 1.15 In the moving-boat method of discharge measurement of rivers the magnitude and direction of the velocity of a stream relative to the moving boat (V_R and θ) are measured. The depth of the stream is also simultaneously recorded. Estimate the discharge in a river (Fig. 1.24) using the following moving-boat data. Assume the velocity to be uniform in a vertical section.

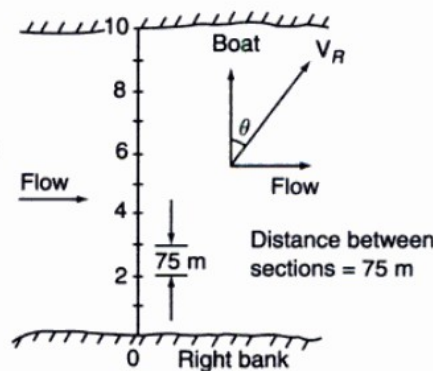


Fig. 1.24 Problem 1.15

Section	V_R (m/s)	θ (degrees)	Depth (m)
1	1.75	55	1.8
2	1.84	57	2.5
3	2.00	60	3.5
4	2.28	64	4.0
5	2.28	64	4.0
6	2.20	63	4.0
7	2.00	60	3.0
8	1.84	57	2.5
9	1.70	54	2.0

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- 1.16 In a rectangular channel, the flow has a free overfall. The velocity measurement at the end section where the flow was curvilinear is indicated in Fig. 1.25. Estimate the discharge per unit width of the channel.

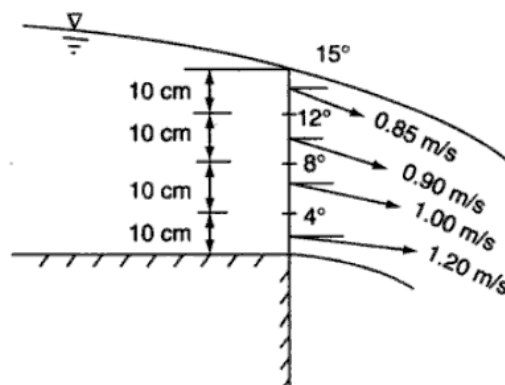


Fig. 1.25 Problem 1.16

- 1.17 Figure 1.26 shows the velocity distribution in a submerged sluice-gate flow. Estimate the discharge per unit width of the gate.

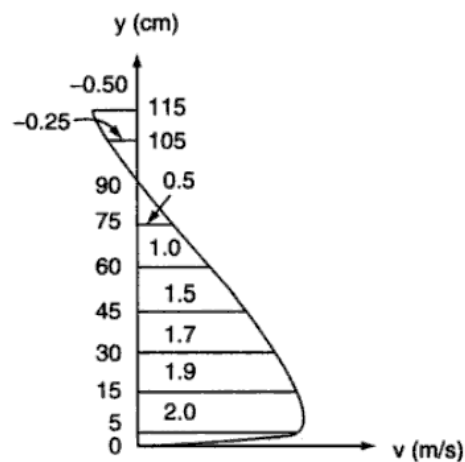


Fig. 1.26 Problem 1.17

- 1.18 A skijump spillway has an exit angle of 40° (Fig. 1.27). If the flow over it has a velocity of 20 m/s, neglecting all losses, estimate the maximum elevation of the outflow trajectory.



Fig. 1.27 Problem 1.18

1.19 Figure 1.28 shows a sluice gate in a rectangular channel. Fill the missing data in the following table:

Case	y_1 (m)	y_2 (m)	q (m ² /s/m)	Losses
(a)	-	0.30	2.5	neglect
(b)	4.0	-	2.0	neglect
(c)	4.0	-	2.0	$0.1 V_2^2/2g$
(d)	3.0	0.25	-	neglect

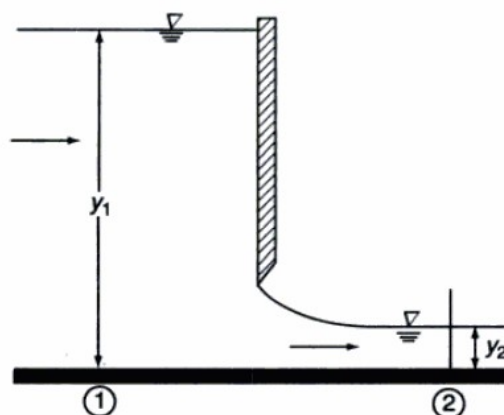


Fig. 1.28 Problem 1.19

- 1.20 A transition in a cross drainage canal works consists of a rectangular canal 2.0-m wide changing into a trapezoidal canal section of 3.0 m bottom width and side slopes 1.5 horizontal: 1 vertical. The depths of flow of 1.5 m in the rectangular section and 1.0 m in the trapezoidal section for a discharge of 10.0 m²/s is envisaged. If a loss of energy = (0.2 x difference of velocity heads) is to be included, calculate the difference in water surface and bed elevations of the two end sections of the transition. Sketch the longitudinal section of the transition, showing the water-surface elevations and the energy line.
- 1.21 Gradually varied flow is found to occur in a channel having an inclination of 10° with the horizontal. At the normal Section A, the elevation of the bed is 15.00 m, the elevation of the water surface is 16.30 m and the velocity of flow is 3.0 m/s. At the normal Section B, the elevation of the bed is 14.60 m and the water surface elevation is 15.80 m. Calculate the elevations of total energy and hydraulic grade lines at normal Sections A and B. Assume the values of the kinetic energy correction factor at A and B as 1.03 and 1.02, respectively.
- 1.22 An expansion in a horizontal rectangular channel takes place from a width of 2.0 m to 3.0 m. The depths of flow for a discharge of 7.20 m³/s are 1.20 m and 1.40 m in the narrower and wider sections respectively. Estimate the energy loss in the transition. Assume the kinetic energy correction coefficient α to have values of 1.05 and 1.15 at the inlet and outlet of the transition, respectively.
- 1.23 Figure 1.29 shows the flow over a spillway. The depths of water are h_1 and h_2 and V_1 is the upstream approach velocity. Estimate the horizontal force on the spillway structure.

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Is the data enough to calculate the vertical component of the force on the spillway also?
If not, what additional information is needed?

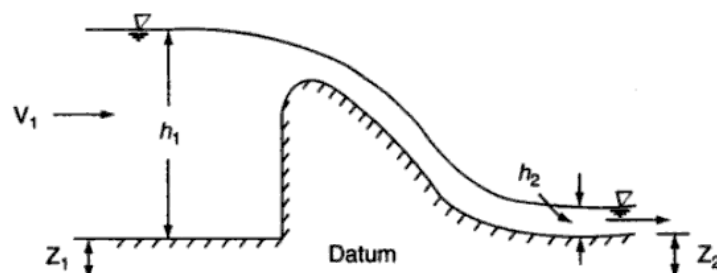


Fig. 1.29 Problem 1.23

- 1.24 Afflux is the net differential water-surface elevation between the upstream and downstream sections of a constriction to the flow. A 15 m wide rectangular canal, has a two-span bridge at a section. The bridge pier is 0.80 m wide and its coefficient of drag C_D is estimated as 2.0. If the depth of flow downstream of the bridge is 2.0 m for a discharge of $80 \text{ m}^3/\text{s}$ in the canal, estimate the afflux due to the bridge.
(Hint: The drag force on the bridge pier = $C_D a \rho V_1^2/2$, where a = projected area of the pier offered to the flow.)
- 1.25 A high-velocity flow from a hydraulic structure has a velocity of 6.0 m/s and a depth of 0.40 m. It is deflected upwards at the end of a horizontal apron through an angle of 45° into the atmosphere as a jet by an end sill. Calculate the force on the sill per unit width.
- 1.26 In Problem 1.19 (a, b, c and d), determine the force per m width of the sluice gate.
- 1.27 Analyse the force on a sluice gate (Example 1.7) when it is discharging under submerged conditions. What additional assumptions are required?
- 1.28 Figure 1.30 shows a submerged flow over a sharp-crested weir in a rectangular channel. If the discharge per unit width is $1.8 \text{ m}^3/\text{s}/\text{m}$, estimate the energy loss due to the weir. What is the force on the weir plate?

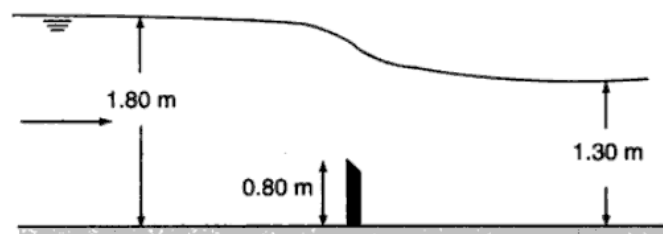


Fig. 1.30 Problem 1.28

- 1.29 A hydraulic jump assisted by a two-dimensional block is formed on a horizontal apron as shown in Fig. 1.31. Estimate the force F_D in kN/m width on the block when a discharge of $6.64 \text{ m}^3/\text{s}$ per m width enters the apron at a depth of 0.5 m and leaves it at a depth of 3.6 m.

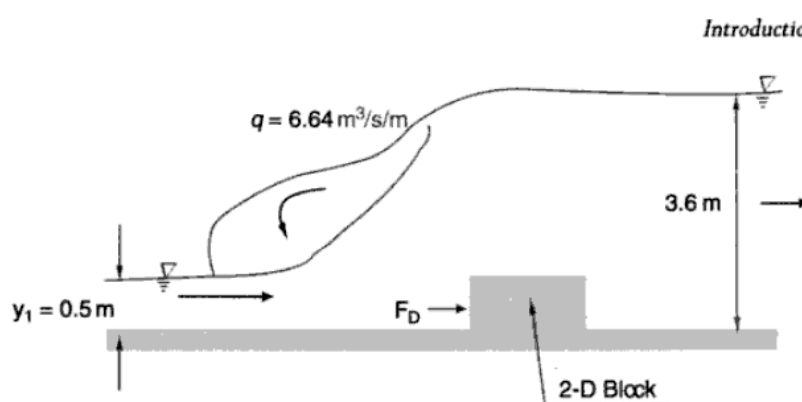


Fig. 1.31 Problem 1.29

- 1.30 Figure 1.32 shows a free overfall in a horizontal frictionless rectangular channel. Assuming the flow to be horizontal at Section 1 and the pressure at the brink of Section 2 to be atmospheric throughout the depth, show that

$$\frac{y_e}{y_0} = \frac{2F_0^2}{(2F_0^2 + 1)}$$

where $F_0^2 = \frac{q^2}{g y_0^3}$ and q = discharge per unit width.



Fig. 1.32 Problem 1.30

- 1.31 Figure 1.33 shows a free overfall at the end of a horizontal, rectangular and frictionless prismatic channel. The space below the lower nappe is fully ventilated. It can be assumed that the water leaves the brink horizontally at a brink depth of y_e . Considering the control volume shown in the figure, show that the back-up depth of water y_1 below the nappe is given by

$$\left[\frac{y_1}{y_2} \right]^2 = 1 + 2F_2^2 \left[1 - \frac{y_2}{y_e} \right]$$

where $F_2 = \frac{q}{y_2 \sqrt{g y_2}}$ and q = discharge per unit width of the channel.

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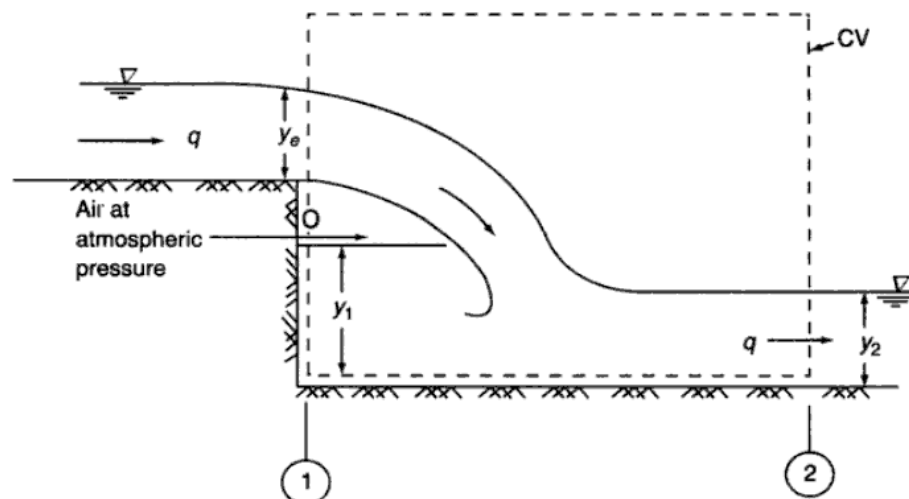


Fig. 1.33 Problem 1.31

OBJECTIVE QUESTIONS

- 1.1 Steady flow in an open channel exists when the
 - (a) flow is uniform
 - (b) depth does not change with time
 - (c) channel is frictionless
 - (d) channel bed is not curved
- 1.2 In a steady spatially-varied flow in a prismatic open channel, the
 - (a) depth does not change along the channel length
 - (b) discharge is constant along its length
 - (c) discharge varies along the length of channel
 - (d) discharge varies with respect to time
- 1.3 A flood wave while passing down a river section protected by embankments, spills over the embankment at certain locations. The flow is classified as
 - (a) steady GVF
 - (b) unsteady RVF
 - (c) steady SVF
 - (d) unsteady SVF
- 1.4 In the uniform flow in a channel of small bed slope, the hydraulic grade line
 - (a) coincides with the bed
 - (b) is considerably below the free surface
 - (c) is considerably above the free surface
 - (d) essentially coincides with the free surface
- 1.5 A uniform flow takes place in a steep channel of large slope. The hydraulic gradient line
 - (a) coincides with the bed
 - (b) essentially coincides with the free surface
 - (c) is above the free surface
 - (d) is below the free surface
- 1.6 One-dimensional method of flow analysis means
 - (a) uniform flow
 - (b) steady uniform flow
 - (c) neglecting the variations in the transverse directions
 - (d) neglecting the variations in the longitudinal direction
- 1.7 At a section in a channel expansion, the velocity over a quarter of the cross-section is zero and is uniform over the remaining three-fourths of the area. The kinetic energy correction factor α is

- (a) 1.78 (b) 1.33 (c) 1.67 (d) 2.00
- 1.8 The velocity distribution in a vertical in a channel gives a rectangular plot when the velocity as abscissa is plotted against height above the bed as ordinate. The kinetic energy correction for this distribution is
- (a) greater than zero but less than unity (c) equal to unity
(b) less than zero (d) greater than unity
- 1.9 The momentum correction factor β is given by β
- (a) $\frac{1}{V^3 A} \int v^3 dA$ (c) $\frac{1}{V^2 A} \int v^2 dA$
(b) $\frac{1}{VA} \int v dA$ (d) $\frac{1}{V^3 A} \int v^2 dA$
- [In the above, V = average velocity = $\frac{1}{A} \int v dA$]
- 1.10 For Question 1.7 above, the momentum correction factor β is
- (a) 2.33 (b) 1.33 (c) 1.67 (d) 1.78
- 1.11 A steep chute is inclined at 45° to the horizontal and carries a flow at a depth of 0.75 m. The pressure at the bed of the chute in N/m^2 is
- (a) 7358 (b) 3679 (c) 5203 (d) 10401
- 1.12 A steep channel has a depth of flow, measured normal to the bed, of h . If the inclination of the channel to the horizontal is θ , the overturning moment of a side wall is
- (a) $\frac{1}{6} \gamma h^3 \cos^4 \theta$ (c) $\frac{1}{6} \gamma h^3 \cos \theta$
(b) $\frac{1}{6} \gamma h^3 \cos^2 \theta$ (d) $\frac{1}{6} \gamma h^3 / \cos \theta$
- 1.13 In an inclined channel the pressure at a depth y is calculated as γy . If this value is to be accurate within 2 per cent of the true value, the maximum inclination of the channel is
- (a) $78^\circ 30'$ (b) $11^\circ 29'$ (c) $11^\circ 22'$ (d) $8^\circ 8'$
- 1.14 Flow takes place over a spillway crest, which can be assumed to be an arc of a circle, at a depth of y_0 . The pressure at any point located on the crest will be
- (a) $= \gamma y_0 \cos \theta$ (c) always zero
(b) $< \gamma y_0 \cos \theta$ (d) always below atmospheric pressure.
- 1.15 A channel with very small value of longitudinal slope S_0 has its water surface parallel to its bed. With the channel bed as the datum, the variation of the piezometric head H_p with distance above the bed y in this channel can be represented by the following:

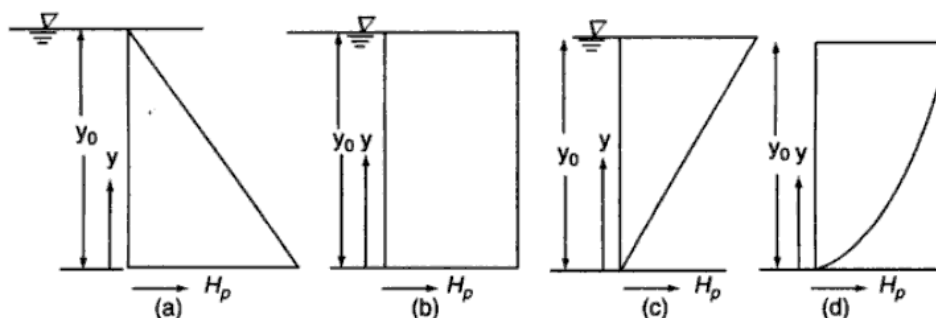


Fig. 1.34 Objective Question 1.15

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- 1.16 A curvilinear flow in a vertical plane has a depth of flow of h and the pressure is found to be uniform at h throughout. The effective piezometric head measured with respect to the bed as the datum is
 (a) $1/2 h$ (b) $1/3 h$ (c) $2/3 h$ (d) $3/2 h$
- 1.17 The velocity and depth of flow in a 3.0 m wide rectangular channel are 2.0 m/s and 2.5 m, respectively. If the channel has its width enlarged to 3.5 m at a section, the discharge past that section is
 (a) 10.0 m³/s (b) 20.0 m³/s (c) 15.0 m³/s (d) 17.5 m³/s
- 1.18 Figure 1.35 shows the velocity distribution at two Sections A and B in a canal. The canal is rectangular in cross section and has widths of 2.0 m at A and 3.5 m at B. Section A is upstream of B. From the data one can infer that
 (a) the discharge in the canal is constant in the reach AB.
 (b) Certain amount of flow is being added into the canal in the reach AB.
 (c) Some amount of flow is being extracted out of the canal in the reach AB.
 (d) The discharge per unit width of the canal is constant in the reach AB.

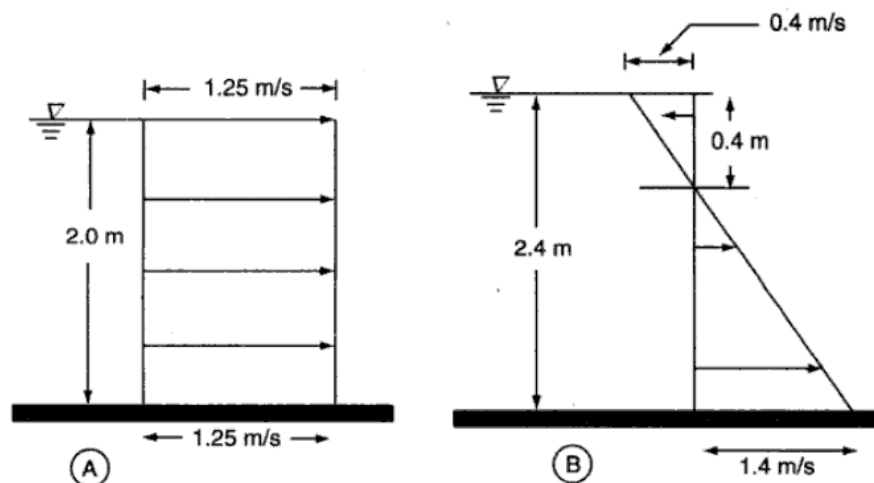


Fig. 1.35 Objective Question 1.18

- 1.19 A sluice gate in a small pond discharges a flow having 10.0 m² flow area and a velocity of 4.0 m/s. If the pond has a surface area of 1.0 hectare, the rate at which the water surface falls in the pond is
 (a) 0.25 m/s (b) 4 cm/s (c) 4 mm/s (d) 4.0 m/s
- 1.20 For an open channel flow to take place between two sections,
 (a) the channel bed must always slope in the direction of the flow
 (b) the upstream depth must be larger than the downstream depth
 (c) the upstream momentum must be larger than the downstream momentum
 (d) the total energy at the upstream end must be larger than the total energy at the downstream section
- 1.21 A steep rectangular channel has a slope of 30° with the horizontal. At a section the bed is 1.20 m above the datum, the depth of flow is 0.70 m the discharge is 3.10 m³/s per metre width. The total energy head at that section by assuming $\alpha = 1.10$ is
 (a) 3.00 m (b) 2.91 m (c) 1.90 m (d) 3.10 m
- 1.22 The width of a rectangular channel is reduced from 3.5 m to 2.5 m at a transition structure. The depth of flow upstream of the contraction is 1.5 m. The change in the bottom elevation required to cause zero change in the water surface elevation is
 (a) -2.1 m (b) -0.6 m (c) +0.6 m (d) -0.2 m

- 1.23 The total energy head for an open channel flow is written with usual notation as $H = z + y + V^2/2g$. In this each of the terms represent
- (a) energy in kg m/kg mass of fluid
 - (b) energy in N m/N of fluid
 - (c) power in kW/kg mass of fluid
 - (d) energy in N m/ mass of fluid
- 1.24 Piezometric head is the sum of
- (a) pressure head, datum head and velocity head
 - (b) datum head and velocity head
 - (c) pressure head and velocity head
 - (d) pressure head and datum head
- 1.25 The difference between total head line and piezometric head line represents
- (a) the velocity head
 - (b) the pressure head
 - (c) the elevation of the bed of the channel
 - (d) the depth of flow
- 1.26 The momentum equation in x -direction as $\Sigma F_x = \rho Q (V_{x2} - V_{x1})$ has the assumption that the flow is
- (a) steady
 - (b) unsteady
 - (c) uniform
 - (d) frictionless
- 1.27 Normally in a stream the ratio of the surface velocity at a location to the average velocity in the vertical through that location
- (a) is greater than 1.0
 - (b) will be between 0.8 and 0.95
 - (c) is less than or greater than unity depending on the type of flow
 - (d) is equal to 0.6
- 1.28 The specific force is constant
- (a) in all frictionless channels irrespective of the magnitude of the longitudinal slope
 - (b) in horizontal, frictionless channels of any shape
 - (c) in all horizontal channels of any shape
 - (d) in any open channel

Energy-Depth Relationships

2

2.1 SPECIFIC ENERGY

The *total energy* of a channel flow referred to a datum is given by Eq.1.39 as

$$H = Z + y \cos \theta + \alpha \frac{V^2}{2g}$$

If the datum coincides with the channel bed at the section, the resulting expression is known as *specific energy* and is denoted by E . Thus

$$E = y \cos \theta + \alpha \frac{V^2}{2g} \quad (2.1)$$

When $\cos \theta = 1.0$ and $\alpha = 1.0$,

$$E = y + \frac{V^2}{2g} \quad (2.2)$$

The concept of specific energy, introduced by Bakhmeteff, is very useful in defining critical depth and in the analysis of flow problems. It may be noted that while the total energy in a real fluid flow always decreases in the downstream direction, the specific energy is constant for a uniform flow and can either decrease or increase in a varied flow, since the elevation of the bed of the channel relative to the elevation of the total energy line, determines the specific energy. If the frictional resistance of the flow can be neglected, the total energy in non-uniform flow will be constant at all sections while the specific energy for such flows, however, will be constant only for a horizontal bed channel and in all other cases the specific energy will vary.

To simplify the expressions it will be assumed, for use in all further analysis, that the specific energy is given by Eq. 2.2, i.e., $\cos \theta = 1.0$ and $\alpha = 1.0$. This is with the knowledge that $\cos \theta$ and α can be appended to y and $(V^2/2g)$ terms respectively, without difficulty if warranted.



2.2 CRITICAL DEPTH

Constant Discharge Situation Since the specific energy

$$E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} \quad (2.2a)$$

for a channel of known geometry, $E = f(y, Q)$ keeping $Q = \text{constant} = Q_1$ the variation of E with y is represented by a cubic parabola Fig. 2.1. It is seen that there are two positive roots for the equation of E indicating that any particular discharge Q_1 can be passed in a given channel at two depths and still maintain the same specific energy E . In Fig. 2.1 the ordinate PP' represents the condition for a specific energy of E_1 . The depths of flow can be either $PR = y_1$ or $PR' = y'_1$. These two possible depths having the same specific energy are known as *alternate depths*. In Fig. 2.1, a line (OS) drawn such that $E = y$ (i.e. at 45° to the abscissa) is the asymptote of the upper limb of the specific energy curve. It may be noticed that the intercept $P'R'$ or $P'R$ represents the velocity head. Of the two alternate depths, one ($PR = y_1$) is smaller and has a large velocity head while the other ($PR' = y'_1$) has a larger depth and consequently a smaller velocity head. For a given Q_1 as the specific energy is increased the difference between the two alternate depths increases. On the other hand, if E is decreased, the difference ($y'_1 - y_1$) will decrease and at a certain value $E = E_c$, the two depths will merge with each other (point C in Fig. 2.1). No value for y can be obtained when $E < E_c$, denoting that the flow under the given conditions is not possible in this region. The condition of minimum specific energy is known as the *critical-flow condition* and the corresponding depth y_c is known as the *critical depth*.

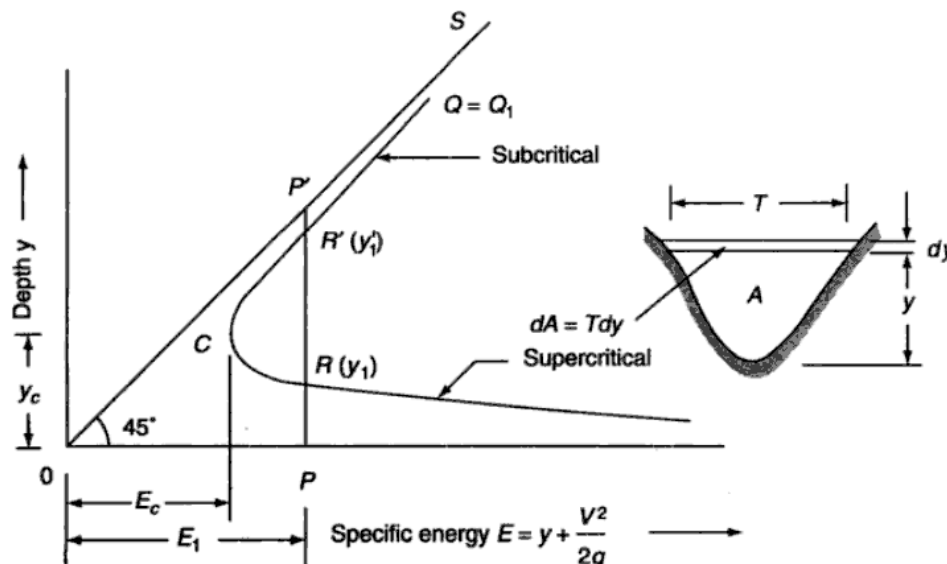


Fig. 2.1 Definition sketch of specific energy

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At critical depth, the specific energy is minimum. Thus differentiating Eq. 2.2a with respect to y (keeping Q constant) and equating to zero,

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3} \frac{dA}{dy} = 0 \quad (2.3)$$

But $\frac{dA}{dy} = T =$ top width, i.e. width of the channel at the water surface.

Designating the critical-flow conditions by the suffix 'c',

$$\frac{Q^2 T_c}{gA_c^3} = 1 \quad (2.4)$$

or
$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} \quad (2.4a)$$

If an α value other than unity is to be used, Eq. 2.4 will become

$$\frac{\alpha Q^2 T_c}{gA_c^3} = 1.0 \quad (2.5)$$

Equation 2.4 or 2.5 is the basic equation governing the critical-flow conditions in a channel. It may be noted that the critical-flow condition is governed solely by the channel geometry and discharge (and α). Other channel properties such as the bed slope and roughness do not influence the critical-flow condition for any given Q . If the Froude number of the flow is define as

$$F = V / (\sqrt{gA/T}) \quad (2.6)$$

it is easy to see that by using F in Eq. 2.4, at the critical flow $y = y_c$ and $F = F_c = 1.0$. We thus get an important result that the critical flow corresponds to the minimum specific energy and at this condition the Froude number of the flow is unity. For a channel with large longitudinal slope θ and having a flow with an energy correction factor of α , the Froude number F will be defined as

$$F = V / \left(\sqrt{\frac{1}{\alpha} g \frac{A}{T} \cos \theta} \right) \quad (2.6a)$$

Referring to Fig. 2.1, considering any specific energy other than E_c , (say ordinate PP' at $E = E_1$) the Froude number of the flow corresponding to both the alternate depths will be different from unity as y_1 or $y'_1 \neq y_c$. At the lower limb, CR of the specific-energy curve, the depth $y_1 < y_c$. As such, $V'_1 > V_c$ and $F_1 > 1.0$. This region is called the *supercritical flow* region. In the upper limb $\bar{C}R'$, $y'_1 > y_c$. As such $V'_1 < V_c$ and $F'_1 < 1.0$. This denotes the *subcritical flow* region.

Discharge as a Variable In the above section the critical-flow condition was derived by keeping the discharge constant. The specific-energy diagram can be plotted

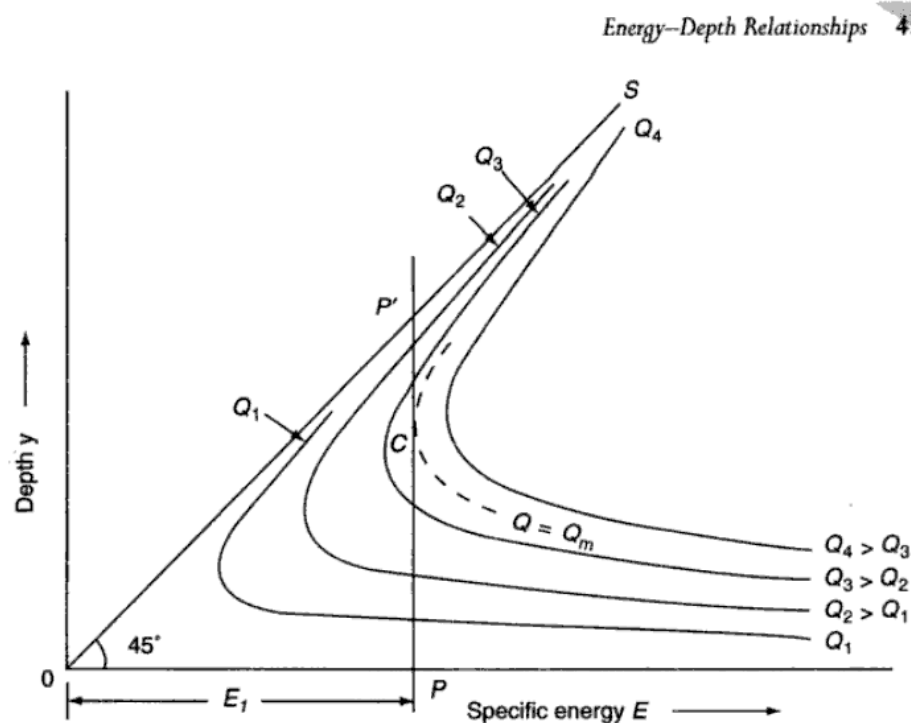


Fig. 2.2 Specific energy for varying discharges

for different discharges $Q = Q_i = \text{constant}$ ($i = 1, 2, 3 \dots$), as in Fig. 2.2. In this figure, $Q_1 < Q_2 < Q_3 < \dots$ and is constant along the respective E vs y plots. Consider a section PP' in this plot. It is seen that for the ordinate PP' , $E = E_1 = \text{constant}$. Different Q curves give different intercepts. The difference between the alternate depths decreases as the Q value increases. It is possible to imagine a value of $Q = Q_m$ at a point C at which the corresponding specific-energy curve would be just tangential to the ordinate PP' . The dotted line in Fig. 2.2 indicating $Q = Q_m$ represents the maximum value of discharge that can be passed in the channel while maintaining the specific energy at a constant value (E_1). Any specific energy curve of higher Q value (i.e. $Q > Q_m$) will have no intercept with the ordinate PP' and hence there will be no depth at which such a discharge can be passed in the channel with the given specific energy. Since by Eq. 2.2a

$$E = y + \frac{Q^2}{2gA^2}$$

$$Q = A\sqrt{2g(E - y)} \quad (2.7)$$

The condition for maximum discharge can be obtained by differentiating Eq. 2.7 with respect to y and equating it to zero while keeping $E = \text{constant}$.

Thus
$$\frac{dQ}{dy} = \sqrt{2g(E - y)} \frac{dA}{dy} - \frac{gA}{\sqrt{2g(E - y)}} = 0$$

By putting $\frac{dA}{dy} = T$ and $\frac{Q}{A} = \sqrt{2g(E - y)}$ yields

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$$\frac{Q^2 T}{g A^3} = 1.0 \quad (2.8)$$

This is same as Eq. 2.4 and hence represents the critical-flow conditions. Hence, the critical-flow condition also corresponds to the condition for maximum discharge in a channel for a fixed specific energy.

Example 2.1 A 2.5-m wide rectangular channel has a specific energy of 1.50 m when carrying a discharge of 6.48 m³/s. Calculate the alternate depths and corresponding Froude numbers.

Solution From Eq. 2.2a

$$\begin{aligned} E &= y + \frac{V^2}{2g} = y + \frac{Q^2}{2gB^2 y^2} \\ 1.5 &= y + \frac{(6.48)^2}{2 \times 9.81 \times (2.5)^2 y^2} \\ &= y + \frac{0.34243}{y^2} \end{aligned}$$

Solving this equation by trial and error, the alternate depths y_1 and y_2 are obtained as $y_1 = 1.296$ m and $y_2 = 0.625$ m.

$$\text{Froude number} \quad F = \frac{V}{\sqrt{gy}} = \frac{6.48}{(2.5y)\sqrt{9.81y}} = \frac{0.82756}{y^{3/2}},$$

At $y_1 = 1.296$ m, $F_1 = 0.561$; and

at $y_2 = 0.625$ m, $F_2 = 1.675$

The depth $y_1 = 1.296$ m is in the subcritical flow region and the depth $y = 0.625$ m is in the supercritical flow region.

Example 2.2 A flow of 5.0 m³/s is passing at a depth of 1.5 m through a rectangular channel of 2.5 m width. The kinetic energy correction factor α is found to be 1.20. What is the specific energy of the flow? What is the value of the depth alternate to the existing depth if $\alpha = 1.0$ is assumed for the alternate flow?

$$\text{Solution} \quad V_1 = \frac{Q}{A_1} = \frac{5.0}{(2.5 \times 1.5)} = 1.33 \text{ m/s}$$

$$\alpha_1 \frac{V_1^2}{2g} = 1.20 \times \frac{(1.33)^2}{2 \times 9.81} = 0.1087 \text{ m}$$

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Specific energy
$$E_1 = y_1 + \alpha_1 \frac{V_1^2}{2g} = 1.5 + 0.1087$$
$$= 1.6087 \text{ m}$$

For the alternate depth y_2 ,

$$y_2 + \frac{(5.0)^2}{2 \times 9.81 (2.5 y_2)^2} = 1.6087 \dots \text{ (since } \alpha_2 = 1.0 \text{)}$$

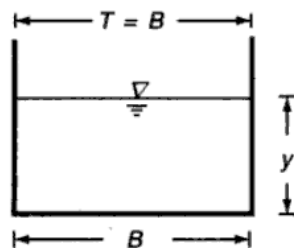
i.e.
$$y_2 + \frac{0.2039}{y_2^2} = 1.6087$$

By trial and error, $y_2 = 0.413 \text{ m}$

2.3 CALCULATION OF THE CRITICAL DEPTH

Using Eq. 2.4, expressions for the critical depth in channels of various geometric shapes can be obtained as follows:

Rectangular Section For a rectangular section, $A = By$ and $T = B$ (Fig. 2.3). Hence by Eq. 2.4



$$\frac{Q^2 T_c}{g A_c^3} = \frac{V_c^2}{g y_c} = 1$$

or
$$\frac{V_c^2}{2g} = \frac{1}{2} y_c \quad (2.9)$$

Fig. 2.3 Rectangular channel

Specific energy at critical depth
$$E_c = y_c + \frac{V_c^2}{2g} = \frac{3}{2} y_c \quad (2.10)$$

Note that Eq. 2.10 is independent of the width of the channel.

Also, if q = discharge per unit width = Q/B ,

$$\frac{q^2}{g} = y_c^3$$

i.e.
$$y_c = \left(\frac{q^2}{g} \right)^{1/3} \quad (2.11)$$

Since $A/T = y$, from Eq. 2.6, the Froude number for a rectangular channel will be defined as

Next available page

$$F = \frac{V}{\sqrt{gy}} \quad (2.12)$$

Triangular Channel For a triangular channel having a side slope of m horizontal: 1 vertical (Fig. 2.4), $A = my^2$ and $T = 2my$.

By Eq. 2.4a,

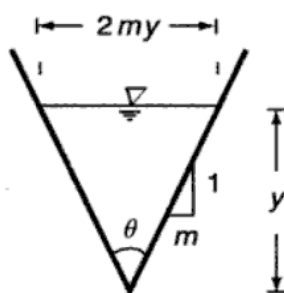


Fig. 2.4 Triangular channel

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{m^3 y_c^6}{2m y_c} = \frac{m^2 y_c^5}{2} \quad (2.13)$$

$$\text{Hence } y_c = \left(\frac{2Q^2}{gm^2} \right)^{1/5} \quad (2.14)$$

The specific energy at critical $E_c = y_c + \frac{V_c^2}{2g}$

$$= y_c + \frac{Q^2}{2gA_c^2} = y_c + \frac{m^2 y_c^5}{4m^2 y_c^4}$$

$$\text{i.e. } E_c = 1.25y_c \quad (2.15)$$

It is noted that Eq. 2.15 is independent of the side slope m of the channel. Since $A/T = y/2$, the Froude number for a triangular channel is defined by using Eq. 2.6 as

$$F = \frac{V\sqrt{2}}{\sqrt{gy}} \quad (2.16)$$

Circular Channel Let D be the diameter of a circular channel (Fig. 2.5) and 2θ be the angle in radians subtended by the water surface at the centre.

A = area of the flow section

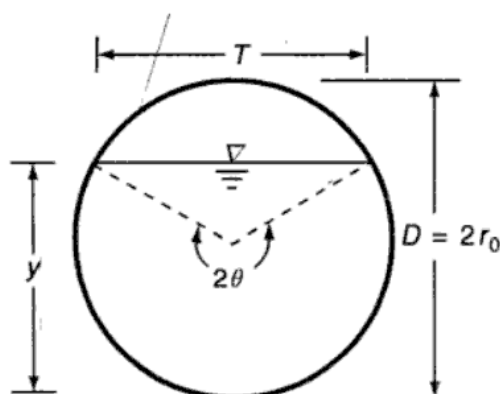


Fig. 2.5 Circular channel

= area of the sector + area of the triangular portion

$$= \frac{1}{2} r_0^2 2\theta + \frac{1}{2} \cdot 2r_0 \sin(\pi - \theta) r_0 \cos(\pi - \theta)$$

$$= \frac{1}{2} (r_0^2 2\theta - r_0^2 \sin 2\theta)$$

$$A = \frac{D^2}{8} (2\theta - \sin 2\theta)$$

$$\text{Top width } T = D \sin \theta$$

$$2\theta = 2 \cos^{-1} \left(1 - \frac{2y}{D} \right) = f(y/D)$$

and

Substituting these in Eq. 2.4a yields

$$\frac{Q^2}{g} = \frac{\left[\frac{D^2}{8} (2\theta_c - \sin 2\theta_c) \right]^3}{D \sin \theta_c} \quad (2.17)$$

Since explicit solutions for y_c cannot be obtained from Eq. 2.17, a non-dimensional representation of Eq. 2.17 is obtained as

$$\frac{Q^2}{\sqrt{gD^5}} = \frac{0.044194 (2\theta_c - \sin 2\theta_c)^{3/2}}{(\sin \theta_c)^{1/2}} = f(y_c/D) \quad (2.18)$$

This function is evaluated and is given in Table 2A.1 of Appendix 2A at the end of this chapter as an aid for the estimation of y_c .

Since $A/T = fn\left(\frac{y}{D}\right)$, the Froude number for a given Q at any depth y will be

$$F = \frac{V}{\sqrt{g(A/T)}} = \frac{Q}{\sqrt{g(A^3/T)}} = fn(y/D)$$

The following are two empirical equations that have been proposed for quick and accurate estimation of critical depth in circular channels:

Empirical relationships for critical depth in circular channels

Sl.No	Equation	Details
1	$\frac{y_c}{D} = [0.77F_D^{-6} + 1.0]^{-0.085}$ where $F_D = \frac{Q}{D^2 \sqrt{gD}} = \frac{Z}{D^{2.5}}$	Swamee P K (1993)(Ref. 4).
2	$y_c = \frac{1.01}{D^{0.265}} \left(\frac{Q}{\sqrt{g}} \right)^{0.506}$ for $0.02 < \frac{y_c}{D} \leq 0.85$	Straub W O (1978)(Ref. 5).

Trapezoidal Channel For a trapezoidal channel having a bottom width of B and side slopes of m horizontal: 1 vertical (Fig. 2.6)

Area $A = (B + my)y$

and Top width $T = (B + 2my)$

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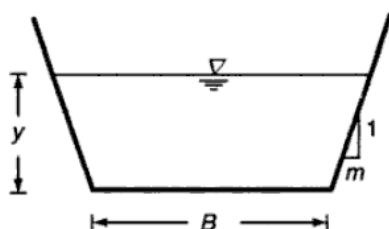


Fig. 2.6 Trapezoidal channel

At the critical flow

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(B + my_c)^3 y_c^3}{(B + 2my_c)} \quad (2.19)$$

Here also an explicit expression for the critical depth y_c is not possible. The non-dimensional representation of Eq. 2.19 facilitates the solution of y_c by the aid of tables or graphs. Rewriting the right-hand side of Eq. 2.19 as

$$\begin{aligned} \frac{(B + my_c)^3 y_c^3}{B + 2my_c} &= \frac{B^3 \left(1 + \frac{my_c}{B}\right)^3 y_c^3}{B \left(1 + \frac{2my_c}{B}\right)} \\ &= \frac{B^5 (1 + \zeta_c)^3 \zeta_c^3}{m^3 (1 + 2\zeta_c)} \quad \text{where } \zeta_c = \frac{m y_c}{B} \end{aligned}$$

gives
$$\frac{Q^2 m^3}{g B^5} = \frac{(1 + \zeta_c)^3 \zeta_c^3}{(1 + 2\zeta_c)} \quad (2.20)$$

or
$$\frac{Q m^{3/2}}{\sqrt{g} B^{5/2}} = \psi = \frac{(1 + \zeta_c)^{3/2} \zeta_c^{3/2}}{(1 + 2\zeta_c)^{1/2}} \quad (2.20a)$$

Equation 2.20a can easily be evaluated for various value of ζ_c and plotted as ψ vs ζ_c . It may be noted that if $\alpha > 1$, ψ can be defined as

$$\psi = \left(\frac{\alpha Q^2 m^3}{g B^5} \right)^{1/2} \quad (2.21)$$

Table 2A - 2 which gives values of ψ for different values of ζ_c is provided at the end of this chapter. This table is very useful in quick solution of problems related to critical depth in trapezoidal channels.

Since $A/T = \frac{(B + my)y}{(B + 2my)} = \frac{\left(1 + \frac{my}{B}\right)y}{\left(1 + 2\frac{my}{B}\right)}$ the Froude number at any depth y is

$$F = \frac{V}{\sqrt{gA/T}} = \frac{Q/A}{\sqrt{gA/T}} = fn (my/B) \text{ for a given discharge } Q.$$

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Further the specific energy at critical depth, E_c is a function of (my_c / B) and it can be shown that (Problem 2.7)

$$\frac{E_c}{y_c} = \frac{1}{2} \frac{(3 + 5\zeta_c)}{(1 + 2\zeta_c)}$$

where

$$\zeta_c = \frac{my_c}{B}$$

2.4 SECTION FACTOR Z

The expression $A\sqrt{A/T}$ is a function of the depth y for a given channel geometry and is known as the *section factor* Z .

Thus
$$Z = A\sqrt{A/T} \quad (2.22)$$

At the critical-flow condition, $y = y_c$ and

$$Z_c = A_c\sqrt{A_c/T_c} = Q/\sqrt{g} \quad (2.23)$$

which is a convenient parameter for analysing the role of the critical depth in a flow problem.

As a corollary of Eq. 2.23, if Z is the section factor for any depth of flow y , then

$$Q_c = \sqrt{g} Z \quad (2.24)$$

where Q_c represents the discharge that would make the depth y critical and is known as the *critical discharge*.

Note that the left-hand side of Eq. 2.18 is a non-dimensional form of the section factor (as $Z/D^{2.5}$) for circular channels.

2.5 FIRST HYDRAULIC EXPONENT M

In many computations involving a wide range of depths in a channel, such as in the GVF computations, it is convenient to express the variation of Z with y in an exponential form.

The $(Z - y)$ relationship

$$Z^2 = C_1 y^M \quad (2.25)$$

is found to be very advantageous. In this equation C_1 is a coefficient and M is an exponent called the *first hydraulic exponent*. It is found that generally M is a slowly-varying

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function of the aspect ratio for most of the channel shapes. The variation of M and $\zeta = \frac{my}{B}$ for a trapezoidal channel is indicated in Fig. 2.7.

The value of M for a given channel can be determined by preparing a plot of Z vs y on a log-log scale. If M is constant between two points (Z_1, y_1) and (Z_2, y_2) in this plot, the value of M is determined as

$$M = 2 \frac{\log(Z_2/Z_1)}{\log(y_2/y_1)} \quad (2.26)$$

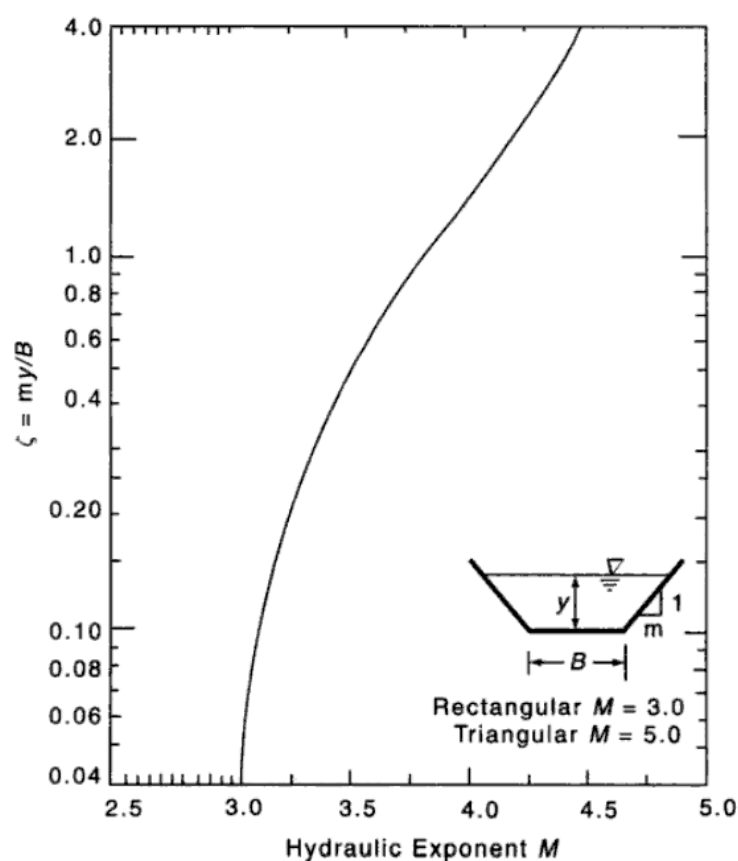


Fig. 2.7 Variation of first hydraulic exponent M in a trapezoidal channel

In Eq. 2.26, instead of Z , a non-dimensionalised Z value can also be used. For a trapezoidal channel, Eq. 2.20a represents a non-dimensionalised value of Z , if the suffix 'c' is removed. Hence the slope of ψ vs my/B on a log-log plot, such as in Fig. 2.7, can be used to obtain the value of M at any value of ζ . It may be noted that M for a trapezoidal channel is a unique function of my/B and will have a value in the range 3.0 to 5.0.



An estimate of M can also be obtained by the relation

$$M = \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right) \quad (2.27)$$

Example 2.3 Obtain the value of the first hydraulic exponent M for (a) rectangular channel, and (b) an exponential channel where the area A is given as $A = K_1 y^a$

Solution (a) For a rectangular channel $A = By$ and $T = B$

$$\text{By Eq. 2.25, } z^2 = \frac{A^2}{T} = B^2 y^3 = C_1 y^M$$

By equating the exponents on both sides, $M = 3.0$

[Note: The above value of M can also be obtained directly by using Eq. 2.27]

$$(b) \quad A = K_1 y^a$$

$$T = \frac{dA}{dy} = K_1 a y^{(a-1)}$$

$$\text{By Eq. 2.27, } M = \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right)$$

$$\begin{aligned} M &= \frac{y}{K_1 y^a} \left[3K_1 a y^{(a-1)} - \frac{K_1 y^a}{K_1 a y^{(a-1)}} \left\{ K_1 a(a-1) y^{(a-2)} \right\} \right] \\ &= 3a - a + 1 = 2a + 1 \end{aligned}$$

Example 2.4 It is required to have a channel in which the Froude number F remains constant at all depths. If the specific energy E is kept constant, show that for

such a channel $\frac{T}{B} = \left[\frac{E}{E-y} \right]^{1+\frac{F^2}{2}}$ where T and B are the top width and bottom width of the channel respectively.

$$\text{Solution} \quad E = y + \frac{V^2}{2g} = y + \frac{Q^2}{2gA^2} = y + \frac{F^2}{2} \left(\frac{A}{T} \right)$$

$$E - y = \frac{AF^2}{2T} \quad (2.28)$$

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Differentiating with respect to y and noting that F is constant,

$$\frac{dE}{dx} - 1 = \frac{F^2}{2} \left[\frac{T \frac{dA}{dy} - A \frac{dT}{dy}}{T^2} \right]$$

Since E is constant, $\frac{dE}{dy} = 0$. Also $\frac{dA}{dy} = T$

Hence,
$$\frac{F^2}{2} \left(1 - \frac{A}{T^2} \frac{dT}{dy} \right) = -1$$

$$\frac{AF^2}{2T} \frac{1}{T} \frac{dT}{dy} = \left(1 + \frac{F^2}{2} \right)$$

Substituting for $\left(\frac{AF^2}{2T} \right)$ from Eq. 2.28, $(E - y) \left(\frac{1}{T} \frac{dT}{dy} \right) = \left(1 + \frac{F^2}{2} \right)$

$$\frac{dT}{T} = \left(1 + \frac{F^2}{2} \right) \frac{dy}{(E - y)}$$

On integration $\ln T = \left(1 + \frac{F^2}{2} \right) (-\ln(E - y)) + C$

At $y = 0$, $T = B$ and hence $C = \ln B + \left(1 + \frac{F^2}{2} \right) \ln E$

$\therefore \ln \frac{T}{B} = \left(1 + \frac{F^2}{2} \right) \ln \left(\frac{E}{E - y} \right)$

or
$$\frac{T}{B} = \left[\frac{E}{E - Y} \right]^{1 + \frac{F^2}{2}}$$

2.6 COMPUTATIONS

The problems concerning critical depth involve the following parameters: geometry of the channel, Q or E or y_c . For rectangular and triangular channel sections, most of the problems involve explicit relationships for the variable and a few problems involve trial and error solutions. However, for trapezoidal, circular and most other regular geometrical shapes of channel sections, many of the problems have to be solved by trial and error procedure. Tables 2A.1 and 2A.2 are helpful



in problems connected with circular and trapezoidal channels respectively. Examples 2.5 to 2.8 illustrate some of the typical problems and the approach to their solutions. The graphical solutions and monographs which were in use some decades back are obsolete now. With the general availability of computers, a large number of elegant numerical methods are available to solve non-linear algebraic equations and the solutions of critical depth and related critical flow problems in channels of all shapes, including natural channels, is no longer difficult.

Example 2.5 Calculate the critical depth and the corresponding specific energy for a discharge of $5.0 \text{ m}^3/\text{s}$ in the following channels:

- (a) Rectangular channel, $B = 2.0 \text{ m}$
- (b) Triangular channel, $m = 0.5$
- (c) Trapezoidal channel, $B = 2.0 \text{ m}$, $m = 1.5$
- (d) Circular channel, $D = 2.0 \text{ m}$

Solution (a) Rectangular Channel

$$q = Q/B = \frac{5.0}{2.0} = 2.5 \text{ m}^3/\text{s/m}$$

$$y_c = (q^2/g)^{1/3} = \left[\frac{(2.5)^2}{9.81} \right]^{1/3} = 0.860 \text{ m}$$

Since for a rectangular channel $\frac{E_c}{y_c} = 1.5$, $E_c = 1.290 \text{ m}$

(b) Triangular Channel

From Eq. 2.14

$$y_c = \left(\frac{2Q^2}{gm^2} \right)^{1/5}$$

$$= \left[\frac{2 \times (5)^2}{9.81 \times (0.5)^2} \right]^{1/5} = 1.828 \text{ m}$$

Since for a triangular channel $\frac{E_c}{y_c} = 1.25$, $E_c = 2.284 \text{ m}$

(c) Trapezoidal Channel

$$\psi = \frac{Qm^{3/2}}{\sqrt{g}B^{5/2}} = \frac{5.0 \times (1.5)^{3/2}}{\sqrt{9.81} \times (2.0)^{5/2}} = 0.51843$$

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Using Table 2A.2 the corresponding value of

$$\zeta_c = \frac{my_c}{B} = 0.536$$

$$y_c = 0.715 \text{ m}$$

$$A_c = (2.0 + 1.5 \times 0.715) \times 0.715 = 2.197 \text{ m}^2$$

$$V_c = 5.0 / 2.197 = 2.276 \text{ m/s}$$

$$V_c^2 / 2g = 0.265 \text{ m}$$

$$E_c = y_c + \frac{V_c^2}{2g} = 0.715 + 0.264 = 0.979 \text{ m}$$

(d) Circular Channel

$$Z_c = \frac{Q}{\sqrt{g}} = \frac{5.0}{\sqrt{9.81}} = 1.5964$$

$$\frac{Z_c}{D^{2.5}} = \frac{1.5964}{(2.0)^{2.5}} = 0.2822$$

From Table 2A.1 showing the relationship of $\frac{Z}{D^{2.5}}$ with y/D , the value of y_c/D corresponding to $\frac{Z_c}{D^{2.5}} = 0.2822$ is found by suitable linear interpolation as $y_c/D = 0.537$ and hence $y_c = 1.074 \text{ m}$.

Determination of y_c by empirical equations**1. By Swamee's equation**

$$F_D = \frac{Q}{D^2 \sqrt{gD}} = \frac{5.0}{(2.0)^2 \sqrt{9.81 \times 2.0}} = 0.2822$$

$$\frac{y_c}{D} = [0.77 F_D^{-6} + 1.0]^{-0.085}$$

$$\frac{y_c}{2.0} = [0.77 (0.2822)^{-6} + 1.0]^{-0.085} = 0.5363$$

$$y_c = 1.072 \text{ m}$$

2. By Straub's equation

$$y_c = \frac{1.01}{D^{0.265}} \left(\frac{Q}{\sqrt{g}} \right)^{0.506}$$



$$y_c = \frac{1.01}{(2.0)^{0.265}} \left(\frac{5.0}{\sqrt{9.81}} \right)^{0.506} = 1.065 \text{ m}$$

Example 2.6 A trapezoidal channel with a bed width of 4.0 m and side slopes of 1.5 H: 1 V carries a certain discharge. (a) Based on observations, if the critical depth of the flow is estimated as 1.70 m, calculate the discharge in the channel. (b) If this discharge is observed to be flowing at a depth of 2.50 m in a reach, estimate the Froude number of the flow in that reach.

Solution (a) At critical depth, area $A_c = (B + my_c) y_c$
 $= [4.0 + (1.5 \times 1.70)] \times 1.70 = 11.135 \text{ m}^2$
 Top width $T_c = (B + 2 m y_c)$
 $= [4.0 + (2 \times 1.5 \times 1.70)] = 9.10 \text{ m}$

At critical flow, by Eq. 2.4a $\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(11.135)^3}{9.10} = 151.715$

Discharge $Q = 38.579 \text{ m}^3/\text{s}$

(b) When the depth of flow $y = 2.50 \text{ m}$
 Area $A = (B + my) y = [4.0 + (1.5 \times 2.50)] \times 2.50 = 19.375 \text{ m}^2$
 Top width $T = (B + 2 my) = [4.0 + (2 \times 1.5 \times 2.50)] = 11.5 \text{ m}$
 $A/T = 19.375/11.50 = 1.685 \text{ m}$
 $V = \frac{Q}{A} = \frac{38.579}{19.375} = 1.991 \text{ m/s}$

Froude number $F = \frac{V}{\sqrt{g(A/T)}} = \frac{1.991}{\sqrt{9.81 \times 1.685}} = 0.490$

Example 2.7 Calculate the bottom width of a channel required to carry a discharge of $15.0 \text{ m}^3/\text{s}$ as a critical flow at a depth of 1.2 m, if the channel section is (a) rectangular, and (b) trapezoidal with side slope 1.5 horizontal: 1 vertical.

Solution (a) Rectangular Section
 The solution here is straightforward.

$$y_c = \left(\frac{q^2}{g} \right)^{1/3} \quad \text{i.e.} \quad q = \sqrt{g y_c^3}$$

$$q = \sqrt{9.81(1.2)^3} = 4.117 \text{ m}^3/\text{s/m}$$

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$$B = \text{bottom width} = \frac{15.0}{4.117} = 3.643 \text{ m}$$

(b) Trapezoidal Channel

The solution in this case is by trial-and-error.

$$A_c = (B + 1.5 \times 1.2) \times 1.2 = (B + 1.8) \times 1.2$$

$$T_c = (B + 2 \times 1.5 \times 1.2) = (B + 3.6)$$

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$\frac{(B + 1.8)^3 \times (1.2)^3}{(B + 3.6)} = \frac{(15)^2}{9.81}$$

$$\frac{(B + 1.8)^3}{(B + 3.6)} = 13.273$$

By trial-and-error

$$B = 2.535 \text{ m}$$

Example 2.8

Find the critical depth for a specific energy head of 1.5 m in the following channels:

- (a) Rectangular channel, $B = 2.0 \text{ m}$
- (b) Triangular channel, $m = 1.5$
- (c) Trapezoidal channel, $B = 2.0 \text{ m}$ and $m = 1.0$
- (d) Circular channel, $D = 1.50 \text{ m}$

Solution (a) *Rectangular Channel*

$$\text{By Eq. 2.10} \quad E_c = \frac{3}{2} y_c = 1.50 \text{ m}$$

$$y_c = \frac{1.50 \times 2}{3} = 1.00 \text{ m}$$

(b) *Triangular Channel*

$$\text{By Eq. 2.15} \quad E_c = 1.25 y_c = 1.50 \text{ m}$$

$$y_c = \frac{1.50}{1.25} = 1.20 \text{ m}$$

(c) *Trapezoidal Channel*

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{Q^2}{2gA_c^3}$$

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Since by Eq. 2.4a $\frac{Q^2}{g} = A_c^3 / T_c$, $E_c = y_c + \frac{A_c}{2T_c}$

$$1.5 = y_c + \frac{(2.0 + y_c)y_c}{2(2.0 + 2y_c)}$$

Solving by trial-and-error, $y_c = 1.095$ m.

(d) *Circular Channel*

$$E_c = y_c + \frac{A_c}{2T_c}$$

By non-dimensionalising with respect to the diameter D .

$$\frac{y_c}{D} + \frac{(A_c/D^2)}{2(T_c/D)} = \frac{E_c}{D} = \frac{1.5}{1.5} = 1.0$$

From Table 2A.1, values of (A_c/D^2) and (T_c/D) for a chosen (y_c/D) are read and a trial-and-error procedure is adopted to solve for y_c/D . It is found that $\frac{y_c}{D} = 0.69$ and $y_c = 0.69 \times 1.50 = 1.035$ m

Example 2.9

Water is flowing a critical depth at a section in a Δ shaped channel, with side slope of 0.5 H: 1 V. (Fig. 2.8). If the critical depth is 1.6 m, estimate the discharge in the channel and the specific energy at the critical depth section.

Solution (i) Here $m = -0.5$

$$T_c = 3.0 - (2 \times 0.5 \times 1.6) = 1.40 \text{ m}$$

$$A_c = \frac{(3.0 + 1.4)}{2} \times 1.60 = 3.52 \text{ m}^2$$

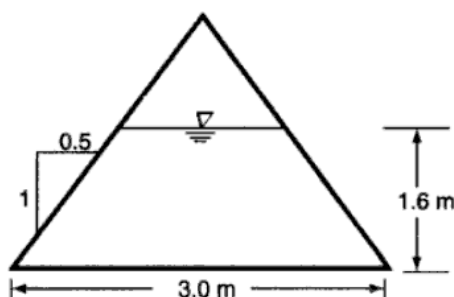


Fig. 2.8 Example 2.9

$$\frac{Q^2}{g} = \frac{A_c^3}{T_c} = \frac{(3.52)^3}{1.40} = 31.153$$

$$\text{Discharge } Q = 17.48 \text{ m}^3/\text{s}$$

$$(ii) \quad V_c = \frac{Q}{A_c} = \frac{17.48}{3.52} = 4.966 \text{ m/s}$$

$$\frac{V_c^2}{2g} = \frac{(4.966)^2}{2 \times 9.81} = 1.257 \text{ m}$$

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$$E_c = y_c + \frac{V_c^2}{2g} = 1.60 + 1.257 = 2.857 \text{ m}$$

2.7 TRANSITIONS

The concepts of specific energy and critical depth are extremely useful in the analysis of problems connected with transitions. To illustrate the various aspects, a few simple transitions in rectangular channels are presented here. The principles are nevertheless equally applicable to channels of any shape and other types of transitions.

2.7.1 Channel with a Hump

(a) Subcritical Flow Consider a horizontal, frictionless rectangular channel of width B carrying Q at a depth y_1 .

Let the flow be subcritical. At Section 2 (Fig. 2.9), a smooth hump of height ΔZ is built on the floor, since there are no energy losses between Sections 1 and 2, and construction of a hump causes the specific energy at Section 2 to decrease by ΔZ . Thus the specific energies at Sections 1 and 2 are given by

$$E_1 = y_1 + \frac{V_1^2}{2g}$$

and

$$E_2 = E_1 - \Delta Z \quad (2.29)$$

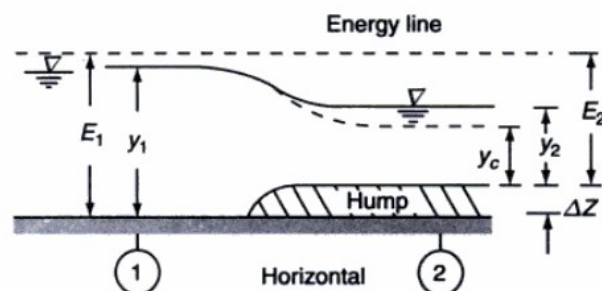


Fig. 2.9 Channel transition with a hump

Since the flow is subcritical, the water surface will drop due to a decrease in the specific energy. In Fig. 2.10, the water surface which was at P at Section 1 will come down to point R at Section 2. the depth y_2 will be given by

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2g B^2 y_2^2} \quad (2.30)$$

It is easy to see from Fig. 2.10 that as the value of ΔZ is increased, the depth at Section 2, i.e. y_2 , will decrease. The minimum depth is reached when the point R coincides with C , the critical depth point. At this point the hump height will be maximum, say $= \Delta Z_m$, $y_2 = y_c =$ critical depth and $E_2 = E_c$. Then condition at ΔZ_m is given by the relation

$$E_1 - \Delta Z_m = E_2 = E_c = y_c + \frac{Q^2}{2g B^2 y_c^2} \quad (2.31)$$

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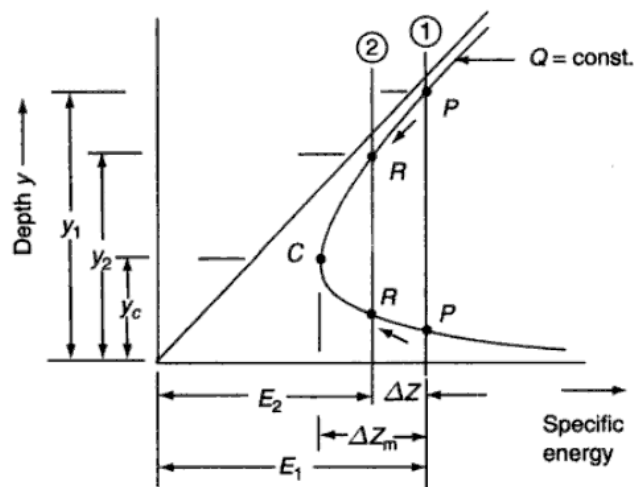


Fig. 2.10 Specific-energy diagram for Fig. 2.9

The question naturally arises as to what happens when $\Delta Z > \Delta Z_m$. From Fig. 2.10 it is seen that the flow is not possible with the given conditions, viz. with the given specific energy. The upstream depth has to increase to cause an increase in the specific energy at Section 1. If this modified depth is represented by y'_1 , then

$$E'_1 = y'_1 + \frac{Q^2}{2gB^2 y_1'^2} \quad \{ \text{with } E'_1 > E_1 \text{ and } y'_1 > y_1 \} \quad (2.32)$$

At Section 2 the flow will continue at the minimum specific energy level, i.e., at the critical condition. At this condition, $y_2 = y_c$ and

$$E'_1 - \Delta Z = E_2 = E_c = y_c + \frac{Q^2}{2gB^2 y_c^2} \quad (2.33)$$

Recollecting the various sequences, when $0 < \Delta Z < \Delta Z_m$ the upstream water level remains stationary at y_1 while the depth of flow at Section 2 decreases with ΔZ reaching a minimum value of y_c at $\Delta Z = \Delta Z_m$ (Fig. 2.11). With further increase in the value of ΔZ , i.e. for $\Delta Z > \Delta Z_m$, y_1 will change to y'_1 while y_2 will continue to remain at y_c .

The variation of y_1 and y_2 with ΔZ in the subcritical regime can be clearly noticed in Fig. 2.11.

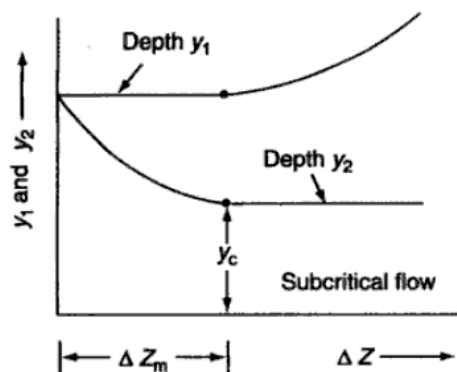


Fig. 2.11 Variation of y_1 and y_2 in subcritical flow over a hump

Minimum Size of Hump for Critical Flow

(i) *Frictionless situation* Consider a smooth, frictionless, streamlined hump of height ΔZ placed at a section in a rectangular channel carrying subcritical flow. The relationship between the specific energies at a section upstream of the hump (E_1) and at section on the hump (E_2) is given as

$$E_1 = E_2 + \Delta Z \quad (2.34)$$

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Let ΔZ_m be the height of the hump that would cause critical flow to occur over the hump (i.e., at Section 2) without changing the upstream specific energy. Any value of $\Delta Z > \Delta Z_m$ would cause critical flow over the hump but the upstream specific energy would change to a value greater than E_1 . Thus ΔZ_m could also be called as the minimum height of a streamlined, frictionless hump that has to be provided to cause critical flow over the hump. An expression for the value of ΔZ_m is obtained as below:

Since at $\Delta Z = \Delta Z_m$, $E_2 = E_c = \frac{3}{2} y_c$ and Eq. 2.28 would now read as

$$E_1 = E_c + \Delta Z_m$$

$$\Delta Z_m = E_1 - \frac{3}{2} y_c$$

$$\frac{\Delta Z_m}{y_1} = \frac{E_1}{y_1} - \frac{3}{2} \frac{y_c}{y_1} = \left(1 + \frac{F_1^2}{2} - \frac{3}{2} \left(\frac{q^2}{g} \right)^{1/3} \frac{1}{y_1} \right)$$

$$\frac{\Delta Z_m}{y_1} = \left(1 + \frac{F_1^2}{2} - \frac{3}{2} F_1^{2/3} \right) \quad (2.35)$$

(ii) When there is energy loss due to the hump Let h_l = energy loss in the transition due to the hump. The energy Eq. 2.34 will now be written as

$$E_1 = E_2 + \Delta Z + h_l \quad (2.36)$$

Following the same procedure as in the frictionless case and noting that $E_2 = E_c$, Eq. 2.35 will be modified as

$$\left(\frac{\Delta Z_m}{y_1} + \frac{h_l}{y_1} \right) = \left(1 + \frac{F_1^2}{2} - \frac{3}{2} F_1^{2/3} \right) \quad (2.37)$$

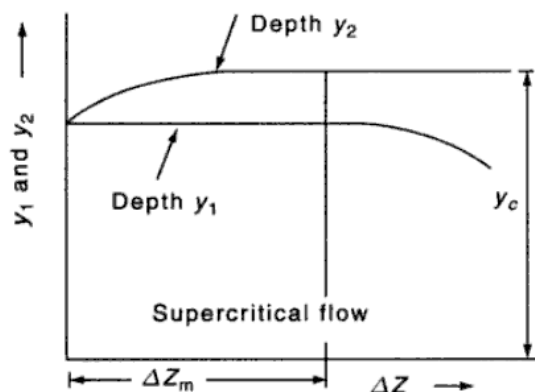


Fig. 2.12 Variation of y_1 and y_2 in supercritical flow over a hump

Comparing Eq. 2.37 with Eq. 2.35, it may be noted that the effect of energy loss in the transition due to shape and friction is equivalent to that of a hump placed in the downstream section.

(b) **Supercritical Flow** If y_1 is in the supercritical flow regime, Fig. 2.12 shows that the depth of flow increases due to the reduction of specific energy. In Fig. 2.10 point P' corresponds to y_1 and

point R' to depth at the Section 2. Up to the critical depth, y_2 increases to reach y_c at $\Delta Z = \Delta Z_m$. For $\Delta Z > \Delta Z_m$, the depth over the hump $y_2 = y_c$ will remain constant and the upstream depth y_1 will change. It will decrease to have a higher specific energy E_1 . The variation of the depths y_1 and y_2 with ΔZ in the supercritical flow is shown in Fig. 2.12.

Example 2.10 *A rectangular channel has a width of 2.0 m and carries a discharge of 4.80 m³/s with a depth of 1.60 m. at a certain section a small, smooth hump with a flat top and of height 0.10 m is proposed to be built. Calculate the likely change in the water surface. Neglect the energy loss.*

Solution Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively as in Fig. 2.9.

$$q = \frac{4.80}{2.0} = 2.40 \text{ m}^3/\text{s/m}$$

$$V_1 = \frac{2.40}{1.6} = 1.50 \text{ m/s}, \quad \frac{V_1^2}{2g} = 0.115 \text{ m}$$

$F_1 = V_1 / \sqrt{gy_1} = 0.379$, hence the upstream flow is subcritical and the hump will cause a drop in the watersurface elevation.

$$E_1 = 1.60 + 0.115 = 1.715 \text{ m}$$

At Section 2,

$$E_2 = E_1 - \Delta Z = 1.715 - 0.10 = 1.615 \text{ m}$$

$$y_c = \left(\frac{(2.4)^2}{9.81} \right)^{1/3} = 0.837 \text{ m}$$

$$E_c = 1.5 y_c = 1.256 \text{ m}$$

The minimum specific energy at Section 2, E_{c2} is less than E_2 , the available specific energy at that section. Hence $y_2 > y_c$ and the upstream depth y_1 will remain unchanged. The depth y_2 is calculated by solving the specific energy relation

$$y_2 + \frac{V_2^2}{2g} = E_2$$

$$\text{i.e.} \quad y_2 + \frac{(2.4)^2}{2 \times 9.81 \times y_2^2} = 1.615$$

Solving by trial-and-error, $y_2 = 1.481 \text{ m}$.

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Example 2.11 (a) In example 2.10, if the height of the hump is 0.5 m, estimate the water surface elevation on the hump and at a section upstream of the hump.
(b) Estimate the minimum size of the hump to cause critical flow over the hump.

Solution (a) From Example 2.10: $F_1 = 0.379$, $E_1 = 1.715$ m and $y_c = y_{c2} = 0.837$ m.

Available specific energy at Section 2 = $E_2 = E_1 - \Delta Z$

$$E_2 = 1.715 - 0.500 = 1.215 \text{ m}$$

$$E'_2 = 1.5 y_{c2} = 1.256 \text{ m.}$$

The minimum specific energy at the Section 2 is greater than E_2 , the available specific energy at that section. Hence, the depth at Section 2 will be at the critical depth. Thus $y_2 = y_{c2} = 1.256$ m. The upstream depth y_1 will increase to a depth y'_1 , such that the new specific energy at the upstream Section 1 is

$$E'_1 = E_{c2} + \Delta Z$$

Thus

$$E'_1 = y'_1 + \frac{V_1'^2}{2g} = E_{c2} + \Delta Z$$

$$y'_1 + \frac{q^2}{2g y_1'^2} = 1.256 + 0.500 = 1.756$$

$$y'_1 + \frac{(2.4)^2}{2 \times 9.81 \times y_1'^2} = 1.756$$

$$y'_1 + \frac{0.2936}{y_1'^2} = 1.756$$

Solving by trial-and-error and selecting the positive root which gives $y'_1 > y_2$,
 $y'_1 = 1.648$ m

The nature of the water surface is shown in Fig. 2.13.

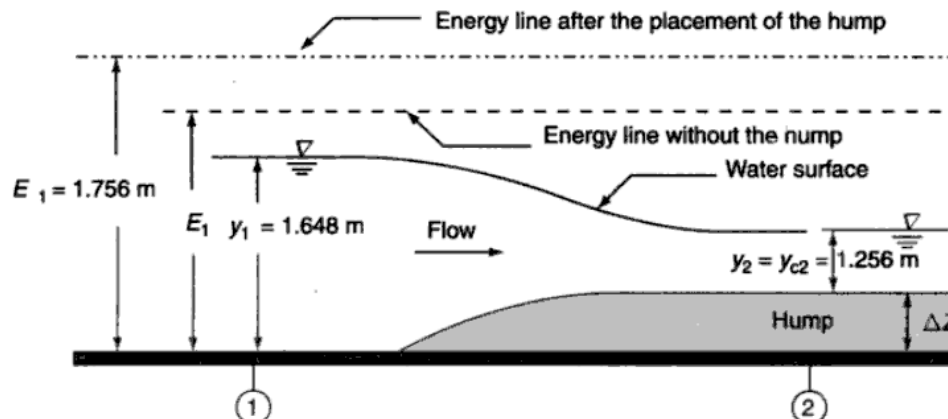


Fig. 2.13 Example 2.11



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(b) Here, $F_1 = 0.379$ and $y_1 = 1.6$ m. By use of Eq. 2.35

$$\frac{\Delta Z_m}{y_1} = \left(1 + \frac{F_1^2}{2} - \frac{3}{2} F_1^{2/3} \right)$$

$$\frac{\Delta Z_m}{1.6} = \left(1 + \frac{(0.379)^2}{2} - \frac{3}{2} (0.379)^{2/3} \right) = 0.2866$$

$$\Delta Z_m = 0.459 \text{ m}$$

Alternatively, $\Delta Z_m = E_1 - E_c = 1.715 - 1.256 = 0.459 \text{ m}$

Example 2.12

A 2.5-m wide rectangular channel carries $6.0 \text{ m}^3/\text{s}$ of flow at a depth of 0.50 m. Calculate the minimum height of a streamlined, flat-topped hump required to be placed at a section to cause critical flow over the hump. The energy loss over the hump can be taken as 10% of the upstream velocity head.

Solution Discharge intensity $q = 6.0/2.5 = 2.40 \text{ m}^3/\text{s}/\text{m}$

$$V_1 = 2.4/0.5 = 4.8 \text{ m/s}, \quad \frac{V_1^2}{2g} = 1.174 \text{ m}$$

$$\text{Energy loss, } h_L = 0.1 \times \frac{V_1^2}{2g} = 0.1174$$

$$\text{Froude number, } F_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{4.8}{\sqrt{9.81 \times 0.5}} = 2.167$$

By Eq. (2.36) $\left(\frac{\Delta Z_m}{y_1} + \frac{h_L}{y_1} \right) = \left(1 + \frac{F_1^2}{2} - \frac{3}{2} F_1^{2/3} \right)$

$$\left(\frac{\Delta Z_m}{0.5} + 0.1174 \right) = \left(1 + \frac{(2.167)^2}{2} - \frac{3}{2} (2.167)^{2/3} \right)$$

$$\frac{\Delta Z_m}{0.5} = 0.6017 \text{ and } \Delta Z_m = 0.301 \text{ m}$$

Alternatively, $\Delta Z_m = E_1 - E_c - h_L$

$$E_c = 1.5 y_c = 1.5 \times \left(\frac{q^2}{g} \right)^{1/3}$$

$$= 1.5 \times \left(\frac{(2.4)^2}{9.81} \right)^{1/3} = 1.256 \text{ m}$$

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$$h_L = 0.1174 \text{ m and } E_1 = y_1 + \frac{V_1^2}{2g} = 0.5 + 1.174 = 1.674 \text{ m}$$

$$\Delta Z_m = 1.674 - 1.256 - 0.1174 = 0.301 \text{ m}$$

2.7.2 Transition with a Change in Width

(a) **Subcritical flow in a Width Constriction** Consider a frictionless horizontal channel of width B_1 carrying a discharge Q at a depth y_1 as in Fig. 2.14. At the Section 2 the channel width has been constricted to B_2 by a smooth transition. Since there are no losses involved and since the bed elevations at Sections 1 and 2 are same, the specific energy at Section 1 is equal to the specific energy at the Section 2.

$$E_1 = y_1 + \frac{V_1^2}{2g} = y_1 + \frac{Q^2}{2gB_1^2 y_1^2}$$

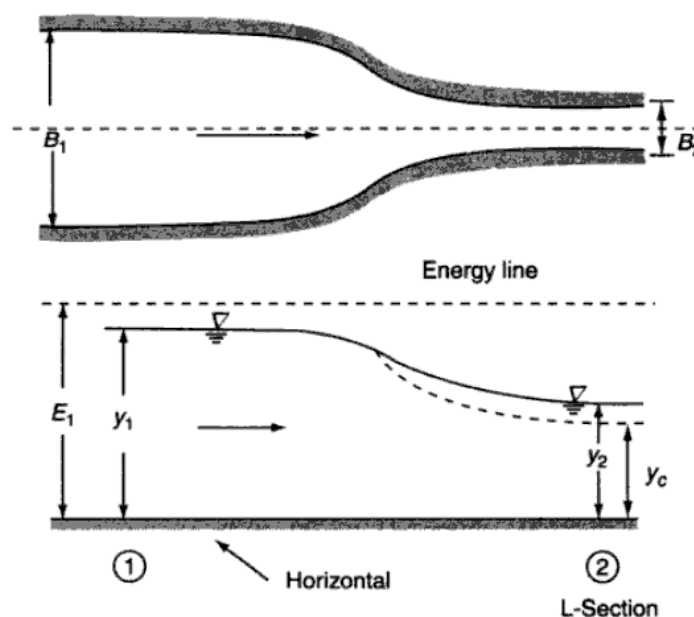


Fig. 2.14 Transition with width constriction

and

$$E_2 = y_2 + \frac{V_2^2}{2g} = y_2 + \frac{Q^2}{2gB_2^2 y_2^2}$$

It is convenient to analyse the flow in terms of the discharge intensity $q = Q/B$. At Section 1, $q_1 = Q/B_1$ and at Section 2, $q_2 = Q/B_2$. Since $B_2 < B_1$, $q_2 > q_1$. In the specific energy diagram (Fig. 2.15) drawn with the discharge intensity as the third parameter, point P on the curve q_1 corresponds to depth y_1 and specific energy E_1 . since at Section 2, $E_2 = E_1$ and $q = q_2$, the point P will move vertically downward to point R on the curve q_2 to reach the depth y_2 . Thus, in subcritical flow the depth

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$y_2 < y_1$. If B_2 is made smaller, then q_2 will increase and y_2 will decrease. The limit of the contracted width $B_2 = B_{2m}$ is obviously reached when corresponding to E_1 , the discharge intensity $q_2 = q_m$, i.e., the maximum discharge intensity for a given specific energy (critical-flow condition) will prevail. At this minimum width, $y_2 =$ critical depth at Section 2, y_{cm} and

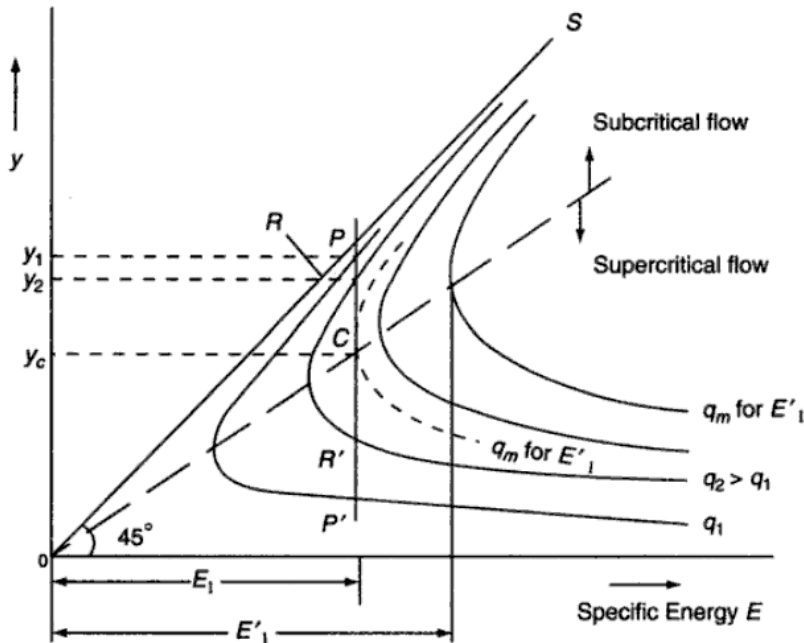


Fig. 2.15 Specific energy diagram for transition of Fig. 2.14

$$E_1 = E_{cm} = y_{cm} + \frac{Q^2}{2g(B_{2m})^2 y_{cm}^2} \quad (2.38)$$

For a rectangular channel, at critical flow $y_c = \frac{2}{3} E_c$

Since $E_1 = E_{cm}$

$$y_2 = y_{cm} = \frac{2}{3} E_{cm} = \frac{2}{3} E_1 \quad (2.39)$$

and $y_c = \left(\frac{Q^2}{B_{2m}^2 g} \right)^{1/3}$ or $B_{2m} = \sqrt{\frac{Q^2}{g y_{cm}^3}}$

i.e. $B_{2m} = \sqrt{\frac{27Q^2}{8gE_1^3}} \quad (2.40)$

If $B_2 < B_{2m}$, the discharge intensity q_2 will be larger than q_m the maximum discharge intensity consistent with E_1 . The flow will not, therefore, be possible with the

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given upstream conditions. The upstream depth will have to increase to y_1 so that a new specific energy $E'_1 = y'_1 + \frac{Q^2}{2g(B_1^2 y_1'^2)}$ is formed which will just be sufficient to cause critical flow at Section 2. It may be noted that the new critical depth at Section 2 for a rectangular channel is

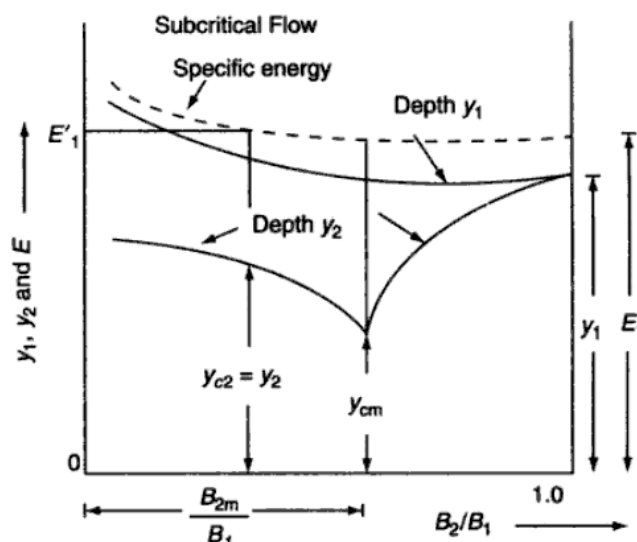


Fig. 2.16 Variation of y_1 and y_2 in subcritical flow in a width constriction

$$y_{c2} = \left[\frac{Q^2}{B_2^2 g} \right]^{1/3} = (q_2^2 / g)^{1/3}$$

and

$$E_{c2} = y_{c2} + \frac{V_{c2}^2}{2g} = 1.5 y_{c2}$$

Since $B_2 < B_{2m}$, y_{c2} will be larger than y_{cm} . Further, $E'_1 = E_{c2} = 1.5 y_{c2}$. Thus even though critical flow prevails for all $B_2 < B_{2m}$, the depth at Section 2 is not constant as in the hump case but increases as y'_1 and hence E'_1 rises. The variation of y_1 , y_2 and E with B_2 / B_1 is shown schematically in Fig. 2.16.

(b) Supercritical Flow in a Width Constriction If the upstream depth y_1 is in the supercritical flow regime, a reduction of the flow width and hence an increase in the discharge intensity causes a rise in depth y_2 . In Fig. 2.15, point P' corresponds to y_1 and point R' to y_2 . As the width B_2 is decreased, R' moves up till it becomes critical at $B_2 = B_{2m}$. Any further reduction in B_2 causes the upstream depth to decrease to y'_1 so that E_1 rises to E'_1 . At Section 2, critical depth y'_c corresponding to the new specific energy E'_1 will prevail. The variation of y_1 , y_2 and E with B_2 / B_1 in supercritical flow regime is indicated in Fig. 2.17.

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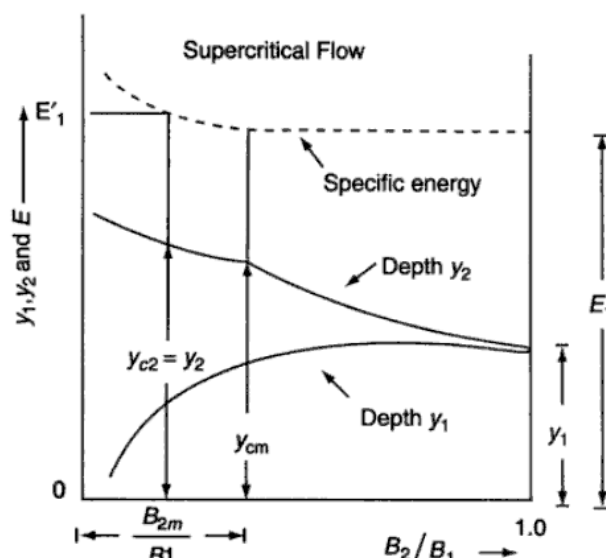


Fig. 2.17 Variation of y_1 and y_2 in supercritical flow in a width constriction

Choking In the case of channel with a hump, and also in the case of a width constriction, it is observed that the upstream water-surface elevation is not affected by the conditions at Section 2 till a critical stage is first achieved. Thus in the case of a hump for all $\Delta Z > \Delta Z_m$, the upstream water depth is constant and for all $\Delta Z > \Delta Z_m$ the upstream depth is different from y_1 . Similarly, in the case of the width constriction, for $B_2 \geq B_{2m}$, the upstream depth y_1 is constant; while for all $B_2 < B_{2m}$, the upstream depth undergoes a change. This onset of critical condition at Section 2 is a prerequisite to choking. Thus all cases with $\Delta Z > \Delta Z_m$ or $B_2 < B_{2m}$ are known as *choked conditions*. Obviously, choked conditions are undesirable and need to be watched in the design of culverts and other surface-drainage features involving channel transitions.

Example 2.13 A rectangular channel is 3.5 m wide and conveys a discharge of 15.0 m³/s at a depth of 2.0 m. It is proposed to reduce the width of the channel at a hydraulic structure. Assuming the transition to be horizontal and the flow to be frictionless determine the water surface elevations upstream and downstream of the constriction when the constricted width is (a) 2.50 m, and (b) 2.20 m.

Solution Let suffixes 1 and 2 denote sections upstream and downstream of the transition respectively. Discharge $Q = B_1 y_1 V_1$

$$V_1 = \frac{15.0}{3.5 \times 2.0} = 2.143 \text{ m/s}$$

$$F_1 = \text{Froude number} = \frac{V_1}{\sqrt{g y_1}} = \frac{2.143}{\sqrt{9.81 \times 2.0}} = 0.484$$

The upstream flow is subcritical and the transition will cause a drop in the water surface.

$$E_1 = y_1 + \frac{V_1^2}{2g} = 2.0 + \frac{(2.143)^2}{2 \times 9.81} = 2.234 \text{ m}$$

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Let B_{2m} = minimum width at Section 2 which does not cause choking.

Then $E_{cm} = E_1 = 2.234 \text{ m}$

$$y_{cm} = \frac{2}{3} E_{cm} = \frac{2}{3} \times 2.234 = 1.489 \text{ m}$$

Since $y_{cm}^3 = \left[\frac{Q^2}{g B_{2m}^2} \right]$

$$B_{2m} = \left[\frac{Q^2}{g y_{cm}^3} \right]^{1/2} = \left[\frac{(15.0)^2}{9.81 \times (1.489)^3} \right]^{1/2} = 2.636 \text{ m}$$

(a) When $B_1 = 2.50 \text{ m}$

$B_2 < B_{2m}$ and hence choking conditions prevail. The depth at the Section 2 = $y_2 = y_{c2}$. The upstream depth y_1 will increase to y'_1 .

Actually $q_2 = \frac{15.0}{2.0} = 6.0 \text{ m}^3/\text{s}/\text{m}$

$$y_{c2} = \left[\frac{q_2^2}{g} \right]^{1/3} = \left[\frac{(6.0)^2}{9.81} \right]^{1/3} = 1.542 \text{ m}$$

$$E_{c2} = 1.5 y_{c2} = 1.5 \times 1.542 = 2.3136 \text{ m}$$

At the upstream Section 1:

$$E'_1 = E_{c2} = 2.3136 \text{ with new upstream depth of } y'_1 \text{ such that } q_1 = y'_1 V'_1 = 15/3.5 = 4.2857 \text{ m}^3/\text{s}/\text{m}.$$

Hence $y'_1 = \frac{V'^2_1}{2g} = 2.3136$

$$y'_1 + \frac{(4.2857)^2}{2 \times 9.81 \times y'^2_1} = 2.3136$$

$$y'_1 + \frac{0.9362}{y'^2_1} = 2.3136$$

Solving by the trial and error and selecting a root that gives subcritical flow,

$$y'_1 = 2.102 \text{ m}$$

(b) When $B_2 = 2.20 \text{ m}$

As $B_2 < B_{2m}$ choking conditions prevail.

Depth at Section 2 = $y_2 = y_{c2}$

$$q_2 = \frac{15.0}{2.20} = 6.8182 \text{ m}^2/\text{s}/\text{m}$$



$$y_{c2} = \left[\frac{(6.8182)^2}{9.81} \right]^{1/3} = 1.6797 \text{ m}$$

$$E_{c2} = 1.5 y_{c2} = 2.5195 \text{ m}$$

At upstream Section 1: New upstream depth = y'_1 and

$$E'_1 = E_{c2} = 2.5195 \text{ m}$$

$$q_1 = V'_1 y'_1 = 15/3.5 = 4.2857 \text{ m}^3/\text{s/m}$$

Hence
$$y'_1 = \frac{V'^2_1}{2g y'_1} = 2.5195$$

$$y'_1 + \frac{(4.2857)^2}{2 \times 9.81 \times y'^2_1} = 2.5195$$

$$y'_1 + \frac{0.9362}{y'^2_1} = 2.5195$$

Solving by trial and error, the appropriate depth to give subcritical flow is

$$y'_1 = 2.350 \text{ m}$$

[Note that for the same discharge when $B_2 < B_{2m}$ (i.e., under choking conditions) the depth at the critical section will be different from y_{cm} and depends on the value of B_2].

2.7.3 General Transition

A transition in its general form may have a change of channel shape, provision of a hump or a depression and contraction or expansion of channel width, in any combination. In addition, there may be various degrees of loss of energy at various components. However, the basic dependence of the depths of flow on the channel geometry and specific energy of flow will remain the same. Many complicated transition situations can be analysed by using the principles of specific energy and critical depth.

In subcritical flow transitions the emphasis is essentially to provide smooth and gradual changes in the boundary to prevent flow separation and consequent energy losses. Details about subcritical flow transitions are available in Ref. 1, 2 and 3. The transitions in supercritical flow, however, are different and involve suppression of shock waves related disturbances and are explained in Chapter 9.

Example 2.14

A discharge of $16.0 \text{ m}^3/\text{s}$ flows with a depth of 2.0 m in a 4.0 m wide rectangular channel. At a downstream section the width is reduced to 3.5 m and the channel bed is raised by ΔZ . Analyse the water-surface elevation in the transitions when (a) $\Delta Z = 0.02 \text{ m}$, and (b) $\Delta Z = 0.35 \text{ m}$.



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Solution Let the suffixes 1 and 2 refer to the upstream and downstream sections respectively.

At the upstream section, $V_1 = \frac{16}{4 \times 2} = 2.0 \text{ m/s}$

$$F_1 = \text{Froude number} = \frac{V_1}{\sqrt{g y_1}} = \frac{2.0}{\sqrt{9.81 \times 2.0}} = 0.452$$

The upstream flow is subcritical and the transition will cause a drop in the water surface elevation.

$$V_1^2/2g = 0.204 \text{ m}$$

$$E_1 = 2.0 + 0.204 = 2.204 \text{ m}$$

q_2 = discharge intensity at the downstream section

$$= \frac{Q}{B_2} = \frac{16.0}{3.5} = 4.571 \text{ m}^3/\text{s/m}$$

y_{c2} = critical depth corresponding to q_2

$$= \left(\frac{q_2^2}{g} \right)^{1/3} = \left[\frac{(4.571)^2}{9.81} \right]^{1/3} = 1.287 \text{ m}$$

$$E_{c2} = \frac{3}{2} y_{c2} = 1.930 \text{ m}$$

(a) When $\Delta Z = 0.20 \text{ m}$

E_2 = available specific energy at Section 2

$$= E_1 - \Delta Z = 2.204 - 0.20 = 2.004 \text{ m} > E_{c2}$$

Hence the depth $y_2 > y_{c2}$ and the upstream depth will remain unchanged at y_1 .

$$y_2 + \frac{V_2^2}{2g} + \Delta Z = E_1$$

$$y_2 + \frac{(4.571)^2}{2 \times 9.81 \times y_2^2} = 2.204 - 0.20$$

$$y_2 + \frac{1.065}{y_2^2} = 2.004$$

Solving by trial and error, $y_2 = 1.575 \text{ m}$.

Hence when $\Delta Z = 0.20 \text{ m}$, $y_1 = 2.00 \text{ m}$ and $y_2 = 1.575 \text{ m}$

(b) When $\Delta Z = 0.35$,

E_2 = available specific energy at Section 2

$$= 2.204 - 0.350 = 1.854 \text{ m} < E_{c2}$$

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Hence the contraction will be working under choked conditions. The upstream depth must rise to create a higher total head. The depth of flow at Section 2 will be critical with $y_2 = y_{c2} = 1.287$ m.

If the new upstream depth is y'_1

$$y'_1 + \frac{Q^2}{2gB_1^2 y_1'^2} = E_{c2} + \Delta Z = 1.930 + 0.350$$

$$y'_1 + \frac{(16)^2}{2 \times 9.81 \times (4.0)^2 \times y_1'^2} = 2.28$$

$$\text{i.e.} \quad y'_1 + \frac{0.8155}{y_1'^2} = 2.280$$

By trail-and-error, $y'_1 = 2.094$ m.

The upstream depth will therefore rise by 0.094 m due to the choked condition at the constriction. Hence, when $\Delta Z = 0.35$ m

$$y'_1 = 2.094 \text{ m and } y_2 = y_{c2} = 1.287 \text{ m.}$$



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5. Straub, W Ö 'A Quick and Easy Way to Calculate Critical and Conjugate Depths in Circular Channels', *Civil Engineering*, Dec. 1978, 99.70–71.



PROBLEMS

Problem Distribution

Topics	Problems
Specific energy and alternate depths	2.1 – 2.7
Minimum specific energy and critical depth	2.8 – 2.16
Computations involving critical depth	2.17 – 2.27; 2.29
First hydraulic exponent, M	2.30 – 2.32
Transitions	
1. with hump/ drop	2.33 – 2.35
2. with change in width	2.36 – 2.38
3. General	2.39 – 2.44
Miscellaneous concepts	2.28

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- 2.1 In a rectangular channel F_1 and F_2 are the Froude numbers corresponding to the alternate depths of a certain discharge. Show that

$$\left(\frac{F_2}{F_1}\right)^{2/3} = \frac{2 + F_2^2}{2 + F_1^2}$$

- 2.2 Show that in a triangular channel, the Froude number corresponding to alternate depths are given by

$$\frac{F_1}{F_2} = \frac{(4 + F_1^2)^{5/2}}{(4 + F_2^2)^{5/2}}$$

- 2.3 A 5.0 m wide rectangular channel carries 20 m³/s of discharge at a depth of 2.0 m. The width beyond a certain section is to be changed to 3.5 m. If it is desired to keep the water-surface elevation unaffected by this change, what modifications are needed to the bottom elevation?
- 2.4 Find the alternate depths corresponding to a specific head of 2.0 m and a discharge of 6.0 m³/s in (a) a trapezoidal channel, $B = 0.9$ m, $m = 1.0$, (b) triangular channel, $m = 1.5$, (c) circular channel, $D = 2.50$ m. (Use the trial and error method. For Part (c) use Table 2A.1)
- 2.5 If y_1 and y_2 are alternate depths in a rectangular channel show that

$$\frac{2y_1^2 y_2^2}{(y_1 + y_2)} = y_c^3$$

and hence the specific energy, $E = \frac{y_1^2 + y_1 y_2 + y_2^2}{(y_1 + y_2)}$

- 2.6 If y_1 and y_2 are the alternate depths in a triangular channel show that

$$\frac{4y_1^4 y_2^4}{(y_1^2 + y_2^2)(y_1 + y_2)} = y_c^5$$

where y_c = critical depth. Show further that the specific energy E is given by

$$\frac{E}{y_1} = \frac{\eta^4 + \eta^3 + \eta^2 + \eta + 1}{(1 + \eta^2)(1 + \eta)}$$

where $\eta = y_2/y_1$

- 2.7 A trapezoidal channel has a bottom width of 6.0 m and side slopes of 1: 1. The depth of flow is 1.5 m at a discharge of 15 m³/s. Determine the specific energy and alternate depth.
- 2.8 Show that for a trapezoidal channel the minimum specific energy E_c is related to the critical depth y_c as

$$E_c = \frac{y_c}{2} \left[\frac{3 + 5\zeta_c}{1 + 2\zeta_c} \right] \quad \text{where } \zeta_c = \frac{m y_c}{B}$$

- 2.9 Prove that the alternate depths in an exponential channel ($A = k y^a$) are given by

$$\frac{2a y_1^{2a} y_2^{2a} (y_1 - y_2)}{(y_1^{2a} - y_2^{2a})} = y_c^{2a+1}$$



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- 2.10 Show that in an exponential channel ($A = k_1 y^a$) the minimum specific energy E_c and the critical depth y_c are related as

$$\frac{E_c}{y_c} = 1 + \frac{1}{2a}$$

- 2.11 A parabolic channel has its profile given by $x^2 = 4ay$. Obtain an expression for the relative specific energy at the critical flow, E_c / y_c for this channel.
- 2.12 Show that for a horizontal frictionless channel the minimum specific force for a specified discharge is obtained at the critical depth.
- 2.13 A channel in which the area is related to the depth as ($A = k_1 y^a$) in which k_1 and a are constants is called an exponential channel. Show that the critical depth for an exponential channel is given by

$$y_c = \left(\frac{Q^2}{g} \frac{a}{k_1^2} \right)^{1/(2a+1)}$$

- 2.14 An exponential channel ($A = k_1 y^a$) carries a flow with a Froude number F_0 at a depth of flow of y_0 . Show that the critical depth y_c is given by

$$\frac{y_c}{y_0} = F_0^x \quad \text{where } x = 2/(2a+1)$$

- 2.15 If it is desired to have a channel in which the flow is critical at all stages, show that the cross section of such a channel is given by.

$$T^2 h^2 = \frac{Q^3}{8g}$$

in which T = top width and h = depth of the water surface below the energy line.

- 2.16 Show that in a parabolic channel ($x = c\sqrt{y}$) the area can be expressed in terms of the top width T as

$$A = \frac{2}{3} Ty$$

Further, show that the critical depth in such a parabolic channel is given by

$$y_c = \left[\frac{27}{8} \frac{Q^2}{g T_c^2} \right]^{1/3} = \left[\frac{27}{32} \frac{Q^2}{g c^2} \right]^{1/4}$$

- 2.17 A channel has a cross section given by the relationship $A = y^{2.5}$. For a critical depth of 0.5 m in this channel, estimate the (i) discharge and (ii) specific energy.
- 2.18 A triangular channel has an apex angle of 60° and carries a flow with a velocity of 2.0 m/s and depth of 1.25 m. (a) Is the flow subcritical or super-critical? (b) What is the critical depth? (c) What is the specific energy? (d) What is the alternate depth possible for this specific energy?
- 2.19 Fill the missing data in the following table connected with critical depth computations in rectangular channels:

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Case	Q (m^3/s)	B (m)	y_c (m)	E_c (m)
(a)	—	3.0	0.50	—
(b)	5.60	—	0.80	—
(c)	7.50	2.5	—	—
(d)	—	2.0	—	0.60

2.20 Fill in the missing data in the following table connected with critical depth computation in triangular channels:

Case	Q (m^3/s)	Side Slope = m	y_c (m)	E_c (m)
(i)	1.50	1.25	—	—
(ii)	—	1.50	0.30	—
(iii)	—	1.00	—	0.60

2.21 Fill in the missing data in the following table relating critical depth in trapezoidal channels:

Case	m	B (m)	y_c (m)	Q (m^3/s)	E_c (m)
(a)	1.5	3.5	—	5.0	—
(b)	2.0	2.0	0.30	—	—
(c)	1.5	—	0.40	2.641	—
(d)	2.0	4.0	—	—	1.111

- 2.22 Calculate the discharges and specific energies corresponding to the following critical depths in circular channels: (a) $y_c = 0.375$ m, $D = 1.50$ m, and (b) $y_c = 0.40$ m, $D = 2.0$ m.
- 2.23 What is the critical depth corresponding to a discharge of 5.0 m^3/s in (a) a trapezoidal channel of $B = 0.80$ m and $m = 1.5$, and (b) a circular channel of $D = 1.50$ m?
- 2.24 In a circular channel of diameter $D = 1.50$ m, the critical depth y_c is known to occur at a specific energy of 1.80 m. Estimate the value of y_c .
- 2.25 A circular channel is to carry a discharge of 558 litres/s. Find the diameter of the conduit such that the flow is critical when the conduit is running quarter full.
- 2.26 A circular culvert of 1.20 -m diameter is flowing half full and the flow is in critical state. Estimate the discharge and the specific energy.
- 2.27 A brick-lined sewer has a semicircular bottom and vertical side walls 0.60 m apart. If the depth of flow at a section where the flow is known to be at a critical state is 0.60 m, estimate the discharge in the sewer.
- 2.28 A rectangular channel section is to have critical flow and at the same time the wetted perimeter is to be minimum. Show that for these two conditions to occur simultaneously, the width of the channel must be equal to $8/9$ times the minimum specific-energy head.

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- 2.29 Water flows in a Δ shaped channel shown in Fig 2.18. Critical depth is known to occur at a section in this canal. Estimate the discharge and specific energy corresponding to an observed critical depth of 1.40 m.

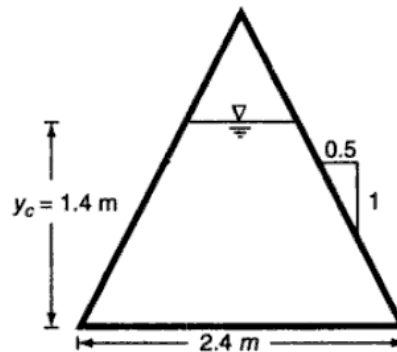


Fig. 2.18 Problem 2.29

- 2.30 For an exponential channel ($A = k_1 y^a$) obtain explicit expressions for the (i) critical depth, (ii) Froude number, (iii) Hydraulic exponent M .
- 2.31 Derive the approximate expression for the first hydraulic exponent M given by Eq. (2.27) as

$$M = \frac{y}{A} \left(3T - \frac{A}{T} \frac{dT}{dy} \right)$$

- 2.32 Estimate the value of the first hydraulic exponent M for the following cases:
- Trapezoidal channel with $B = 4.0$ m, $m = 2.0$ and $y = 0.20$; 1.0 ; 2.0 ; and 4.0 m. [Hint: Use Eq. (2.27)].
 - Circular channel of diameter $D = 2.0$ m with $y = 0.20$; 0.8 ; and 1.6 m. [Hint: Use Eq. (2.27) and Table 2A.1]
- 2.33 A 5.0-m wide rectangular channel carries a discharge of $6.40 \text{ m}^3/\text{s}$ at a depth of 0.8 m. At a section there is a smooth drop of 0.22 m in the bed. What is the water surface elevation downstream of the drop?
- 2.34 A rectangular channel is 4.0 m wide and carries a discharge of $20 \text{ m}^3/\text{s}$ at a depth of 2.0 m. At a certain section it is proposed to build a hump. Calculate the water surface elevations at upstream of the hump and over the hump if the hump height is (a) 0.33 m and (b) 0.20 m. (Assume no loss of energy at the hump.)
- 2.35 A uniform flow of $12.0 \text{ m}^3/\text{s}$ occurs in a long rectangular channel of 5.0 m width and depth of flow of 1.50 m. A flat hump is to be built at a certain section. Assuming a loss of head equal to the upstream velocity head, compute the minimum height of the hump to provide critical flow. What will happen (a) if the height of the hump is higher than the computed value and (b) if the energy loss is less than the assumed value?
- 2.36 A rectangular channel is 3.0 m wide and carries a flow of $1.85 \text{ m}^3/\text{s}$ at a depth of 0.50 m. A contraction of the channel width is required at a certain section. Find the greatest allowable contraction in the width for the upstream flow to be possible as specified.
- 2.37 A rectangular channel is 2.5 m wide and conveys a discharge of $2.75 \text{ m}^3/\text{s}$ at a depth of 0.90 m. A contraction of width is proposed at a section in this canal. Calculate the water surface elevations in the contracted section as well as in an upstream 2.5 m wide section when the width of the proposed contraction is (a) 2.0 m and (b) 1.5 m. (Neglect energy losses in the transition).

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- 2.38 A 3.0-m wide horizontal rectangular channel is narrowed to a width of 1.5 m to cause critical flow in the contracted section. If the depth in the contracted section is 0.8 m, calculate the discharge in the channel and the possible depths of flow and corresponding Froude numbers in the 3.0 m wide section. Neglect energy losses in the transition.
- 2.39 A rectangular channel is 3.0 m wide and carries a discharge of $15.0 \text{ m}^3/\text{s}$ at a depth of 2.0 m. At a certain section of the channel it is proposed to reduce the width to 2.0 m and to alter the bed elevation by ΔZ to obtain critical flow at the contracted section without altering the upstream depth. What should be the value of ΔZ ?
- 2.40 Water flows at a velocity of 1.0 m/s and a depth of 2.0 m in an open channel of rectangular cross section and bed-width of 3.0 m. At certain section the width is reduced to 1.80 m and the bed is raised by 0.65 m. Will the upstream depth be affected and if so, to what extent?
- 2.41 A 3-m wide rectangular channel carries $3 \text{ m}^3/\text{s}$ of water at a depth of 1.0 m. If the width is to be reduced to 2.0 m and bed raised by 10 cm, what would be the depth of flow in the contracted section? Neglect the loss of energy in transition. What maximum rise in the bed level of the contracted section is possible without affecting the depth of flow upstream of the transition?
- 2.42 Water flows in a 3.0-m wide rectangular channel at a velocity of 2.5 m/s and a depth of 1.8 m. If at a section there is a smooth upward step of 0.30 m, what width is needed at that section to enable the critical flow to occur on the hump without any change in the upstream depth?
- 2.43 A 3.0-m wide rectangular channel carries a flow at 1.25 m depth. At a certain section the width is reduced to 2.5 m and the channel bed raised by 0.20 m through a streamlined hump. (a) Estimate the discharge in the channel when the water surface drops by 0.15 m over the hump. (b) What change in the bed elevation at the contracted section would make the water surface have the same elevation upstream and downstream of the contraction? (The energy losses in the contraction can be neglected).
- 2.44 A 1.5-m wide rectangular channel carries a discharge of $5.0 \text{ m}^3/\text{s}$ at a depth of 1.5 m. At a section the channel undergoes transition to a triangular section of side slopes 2 horizontal: 1 vertical. If the flow in the triangular section is to be critical without changing the upstream water surface, find the location of the vertex of the triangular section relative to the bed of the rectangular channel. What is the drop/rise in the water surface at the transition? Assume zero energy loss at the transition.

§ OBJECTIVE QUESTIONS

- 2.1 The term *alternate depths* is used in open channel flow to denote the depths
 - (a) having the same kinetic energy for a given discharge
 - (b) having the same specific force for a given discharge
 - (c) having the same specific energy for a given discharge
 - (d) having the same total energy for a given discharge
- 2.2 The two alternate depths in a 4.0 m wide rectangular channel are 3.86 m and 1.0 m respectively. The discharge in the channel in m^3/s is
 - (a) 15
 - (b) 1.5
 - (c) 7.76
 - (d) 31.0
- 2.3 In a rectangular channel, the alternate depths are 1.0 m and 2.0 m respectively. The specific energy head in m is
 - (a) 3.38
 - (b) 1.33
 - (c) 2.33
 - (d) 3.0

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2.4 A rectangular channel carries a certain flow for which the alternate depths are found to be 3.0 m and 1.0 m. The critical depth in m for this flow is

- (a) 2.65 (b) 1.65 (c) 0.65 (d) 1.33

2.5 The critical flow condition in a channel is given by

(a) $\frac{\alpha Q T^2}{g A^3} = 1$ (b) $\frac{\alpha Q^2 T^2}{g A^3} = 1$

(c) $\frac{\alpha Q^2 T}{g A^3} = 1$ (d) $\frac{\alpha Q T}{g A^3} = 1$

2.6 In defining a Froude number applicable to open channels of any shape, the length parameter used is the

- (a) ratio of area to top width (c) depth of flow
(b) ratio of area to wetted perimeter (d) square root of the area

2.7 For a triangular channel of side slopes m horizontal : 1 vertical, the Froude number is given by $F =$

(a) $\frac{m}{\sqrt{g y}}$ (b) $\frac{V}{\sqrt{2 g y}}$ (c) $\frac{V \sqrt{2}}{g y}$ (d) $\frac{V}{\sqrt{g y}}$

2.8 A triangular channel has a vertex angle of 90° and carries a discharge of $1.90 \text{ m}^3/\text{s}$ at a depth of 0.8 m. The Froude number of the flow is

- (a) 0.68 (b) 1.06 (c) 0.75 (d) 1.50

2.9 A triangular channel of apex angle of 120° carries a discharge of 1573 l/s . The critical depth in m is

- (a) 0.600 (b) 0.700 (c) 0.800 (d) 0.632

2.10 A triangular channel of apex angle of 60° has a critical depth of 0.25 m. The discharge in l/s is

- (a) 60 (b) 640 (c) 160 (d) 40

2.11 At critical depth

- (a) the discharge is minimum for a given specific energy
(b) the discharge is maximum for a given specific force
(c) the discharge is maximum for a given specific energy
(d) the discharge is minimum for a given specific force

2.12 For a given open channel carrying a certain discharge the critical depth depends on

- (a) the geometry of the channel (c) the roughness of the channel
(b) the viscosity of water (d) the longitudinal slope of the channel

2.13 In a triangular channel the value of E_c / y_c is

- (a) 1.25 (b) 2.5 (c) 3.33 (d) 1.5

2.14 In a parabolic channel ($x^2 = 4ay$) the value of E_c / y_c is

- (a) 1.5 (b) 2.0 (c) 3.33 (d) 1.25

2.15 In an exponential channel (Area $A = k y^a$) the ratio of specific energy at critical depth E_c to the critical depth y_c is

(a) $\left[\frac{2a+1}{2a} \right]$ (b) $2a$ (c) $\frac{a+1}{a}$ (d) $\frac{3}{2}a$

2.16 In a rectangular channel carrying uniform flow with a specific energy E and depth of flow $= y_0$ the ratio E / y_c is equal to

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$$(a) \left(\frac{y_0}{y_c} \right)^2 + \frac{1}{2(y_0/y_c)^2} \quad (b) \left(\frac{y_c}{y_0} \right) + 2 \left(\frac{y_c}{y_0} \right)^2$$

$$(c) 2 \left(\frac{y_c}{y_0} \right) + \frac{1}{2(y_c/y_0)^2} \quad (d) \left(\frac{y_0}{y_c} \right) + \frac{1}{2(y_0/y_c)^2}$$

2.17 If F_0 = Froude number of flow in rectangular channel at a depth of flow = y_0 then $y_c/y_0 =$

(a) $\sqrt{F_0}$ (b) $F_0^{2/3}$ (c) $F_0^{1/3}$ (d) $F_0^{3/2}$

2.18 Supercritical flow at Froude number of $F_0 = 2.0$ occurs at a depth of 0.63 m in a rectangular channel. The critical depth in m is

(a) 0.857 (b) 0.735 (c) 1.000 (d) 0.500

2.19 The Froude number of a flow in a rectangular channel is 0.73. If the depth of flow is 1.50 m, the specific energy in metres is

(a) 1.90 (b) 1.50 (c) 1.73 (d) 0.73

2.20 A trapezoidal channel of bed width of 3.5 m and side slope of 1.5 H: 1 V carries a flow of 9.0 m³/s with a depth of 2.0 m. The Froude number of flow is

(a) 0.156 (b) 0.189 (c) 0.013 (d) 0.506

2.21 For a triangular channel the first hydraulic exponent M is

(a) 2.0 (b) 3.0 (c) 5.0 (d) 5.33

2.22 For a trapezoidal canal section with side slope of m horizontal : 1 vertical the value of the first hydraulic exponent M is

- (a) a constant at all stages
(b) a function of S_0 and Manning's coefficient n
(c) a function my/B
(d) a function of y/B only

2.23 In a rectangular channel with subcritical flow the height of a hump to be built to cause subcritical flow over it was calculated by neglecting energy losses. If, after building the hump, it is found that the energy losses in the transition are appreciable, the effect of this hump on the flow will be

- (a) to make the flow over the hump subcritical
(b) to make the flow over the hump supercritical
(c) to cause the depth of flow upstream of the hump to raise
(d) to lower the upstream water surface

2.24 For an exponential channel ($A = ky^a$) the first hydraulic exponent M is

(a) $(a + 1)$ (b) $(2a)$ (c) $(2a + 1)$ (d) a^2

2.25 The flow in a rectangular channel is subcritical. If the width is expanded at a certain section, the water surface

- (a) at a downstream section will drop
(b) at the downstream section will rise
(c) at the upstream section will rise
(d) at the upstream section will drop

2.26 A bottom rack in a channel is used to withdraw a part of the discharge flowing in a canal. If the flow is subcritical throughout, this will cause

- (a) a rise in the water surface on the rack
(b) a drop in the water surface over the rack
(c) a jump over the rack
(d) a lowering of the water surface upstream of the rack.



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2.27 The flow in a channel is at critical depth. If at a section M a small hump of height ΔZ is built on the bed of the channel, the flow will be

- (a) critical upstream of M (b) critical at M
(c) subcritical at M (d) supercritical at M



APPENDIX 2A

Two tables which are useful in various open channel flow computations are presented here.

1. Table 2A.1 contains the geometric elements of a circular channel in a non-dimensional fashion. The column $(Z/D^{2.5}) = f(y/D)$ is useful in calculating critical depths by using Eq. (2.18). At critical depth, $y = y_c$ and

$$(Z/D^{2.5}) = (Q/\sqrt{g}D^{2.5}).$$

The last column, $AR^{2/3}/D^{8/3}$, will be useful in the calculation of normal depths as will be explained in Chapter 3 (Eq. (3.29)). At normal depth,

$$\phi(y_0/D) = \frac{AR^{2/3}}{D^{8/3}} = \frac{Qn}{\sqrt{S_0} D^{8/3}}$$

2. Table 2A.2 contains the values of $\psi = f(\zeta_c)$ for computation of critical depth in trapezoidal channels. At the critical depth y_c'

$$\psi = \left(\frac{\alpha Q^2 m^3}{g B^5} \right)^{1/2} \quad \text{and} \quad \zeta_c = \frac{m y_c}{B}$$

Table 2A.1 Elements of Circular Channels*

y/D	$2\theta(\text{radians})$	A/D^2	P/D	T/D	$Z/D^{2.5}$	$AR^{2/3}/D^{8/3}$
0.01	0.40067 E+00	0.13293 E-02	0.20033 E+00	0.19900 E+00	0.10865 E-03	0.46941 E-04
0.02	0.56759 E+00	0.37485 E-02	0.28379 E+00	0.28000 E+00	0.43372 E-03	0.20946 E-03
0.03	0.69633 E+00	0.68655 E-02	0.34817 E+00	0.34117 E+00	0.97392 E-03	0.50111 E-03
0.04	0.80543 E+00	0.10538 E-01	0.40272 E+00	0.39192 E+00	0.17279 E-02	0.92878 E-03
0.05	0.90205 E+00	0.14681 E-01	0.45103 E+00	0.43589 E+00	0.26944 E-02	0.14967 E-02
0.06	0.98987 E+00	0.19239 E-01	0.49493 E+00	0.47497 E+00	0.38721 E-02	0.22078 E-02
0.07	0.10711 E+01	0.24168 E-01	0.53553 E+00	0.51029 E+00	0.52597 E-02	0.30636 E-02
0.08	0.11470 E+01	0.29435 E-01	0.57351 E+00	0.54259 E+00	0.68559 E-02	0.40652 E-02
0.09	0.12188 E+01	0.35012 E-01	0.60939 E+00	0.57236 E+00	0.86594 E-02	0.52131 E-02
0.10	0.12870 E+01	0.40875 E-01	0.64350 E+00	0.60000 E+00	0.10669 E-01	0.65073 E-02
0.11	0.13523 E+01	0.47006 E-01	0.67613 E+00	0.62578 E+00	0.12883 E-01	0.79475 E-02
0.12	0.14150 E+01	0.53385 E-01	0.70748 E+00	0.64992 E+00	0.15300 E-01	0.95329 E-02
0.13	0.14755 E+01	0.59999 E-01	0.73773 E+00	0.67261 E+00	0.17920 E-01	0.11263 E-01
0.14	0.15340 E+01	0.66833 E-01	0.76699 E+00	0.69397 E+00	0.20740 E-01	0.13136 E-01
0.15	0.15908 E+01	0.73875 E-01	0.79540 E+00	0.71414 E+00	0.23760 E-01	0.15151 E-01
0.16	1.64607	0.08111	0.82303	0.73321	0.02698	0.01731
0.17	1.69996	0.08854	0.84998	0.75127	0.03039	0.01960
0.18	1.75260	0.09613	0.87630	0.76837	0.03400	0.02203



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Table 2A.1 (Continued)

y/D	$2\theta(\text{radians})$	A/D^2	P/D	T/D	$Z/D^{2.5}$	$AR^{2.5}/D^{8/3}$
0.19	1.80411	0.10390	0.90205	0.78460	0.03781	0.02460
0.20	1.85459	0.11182	0.92730	0.80000	0.04181	0.02729
0.21	1.90414	0.11990	0.95207	0.81462	0.04600	0.03012
0.22	1.95282	0.12811	0.97641	0.82849	0.05038	0.03308
0.23	2.00072	0.13647	1.00036	0.84167	0.05495	0.03616
0.24	2.04789	0.14494	1.02395	0.85417	0.05971	0.03937
0.25	2.09440	0.15355	1.04720	0.86603	0.06465	0.04270
0.26	2.14028	0.16226	1.07014	0.87727	0.06979	0.04614
0.27	2.18560	0.17109	1.09280	0.88792	0.07510	0.04970
0.28	2.23040	0.18002	1.11520	0.89800	0.08060	0.05337
0.29	2.27470	0.18905	1.13735	0.90752	0.08628	0.05715
0.30	2.31856	0.19817	1.15928	0.91652	0.09215	0.06104
0.31	2.36200	0.20738	1.18100	0.92499	0.09819	0.06503
0.32	2.40506	0.21667	1.20253	0.93295	0.10441	0.06912
0.33	2.44776	0.22603	1.22388	0.94043	0.11081	0.07330
0.34	2.49013	0.23547	1.24507	0.94742	0.11739	0.07758
0.35	2.53221	0.24498	1.26610	0.95394	0.12415	0.08195
0.36	2.57400	0.25455	1.28700	0.96000	0.13108	0.08641
0.37	2.61555	0.26418	1.30777	0.96561	0.13818	0.09095
0.38	2.65686	0.27386	1.32843	0.97077	0.14546	0.09557
0.39	2.69796	0.28359	1.34898	0.97550	0.15291	0.10027
0.40	2.73888	0.29337	1.36944	0.97980	0.16053	0.10503
0.41	2.77962	0.30319	1.38981	0.98367	0.16832	0.10987
0.42	2.82021	0.31304	1.41011	0.98712	0.17629	0.11477
0.43	2.86067	0.32293	1.43033	0.99015	0.18442	0.11973
0.44	2.90101	0.33284	1.45051	0.99277	0.19272	0.12475
0.45	2.94126	0.34278	1.47063	0.99499	0.20120	0.12983
0.46	2.98142	0.35274	1.49071	0.99679	0.20984	0.13495
0.47	3.02152	0.36272	1.51076	0.99820	0.21865	0.14011
0.48	3.06157	0.37270	1.53079	0.99920	0.22763	0.14532
0.49	3.10159	0.38270	1.55080	0.99980	0.23677	0.15057
0.50	3.14159	0.39270	1.57080	1.00000	0.24609	0.15584
0.51	3.18160	0.40270	1.59080	0.99980	0.25557	0.16115
0.52	3.22161	0.41269	1.61081	0.99920	0.26523	0.16648
0.53	3.26166	0.42268	1.63083	0.99820	0.27505	0.17182
0.54	3.30176	0.43266	1.65088	0.99679	0.28504	0.17719
0.55	3.34193	0.44262	1.67096	0.99499	0.29521	0.18256
0.56	3.38217	0.45255	1.69109	0.99277	0.30555	0.18794
0.57	3.42252	0.46247	1.71126	0.99015	0.31606	0.19331
0.58	3.46297	0.47236	1.73149	0.98712	0.32675	0.19869
0.59	3.50357	0.48221	1.75178	0.98367	0.33762	0.20405
0.60	3.54431	0.49203	1.77215	0.97980	0.34867	0.20940
0.61	3.58522	0.50181	1.79261	0.97550	0.35991	0.21473
0.62	3.62632	0.51154	1.81316	0.97077	0.37133	0.22004
0.63	3.66764	0.52122	1.83382	0.96561	0.38294	0.22532



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y/D	$2\theta(\text{radians})$	A/D^2	P/D	T/D	$Z/D^{2.5}$	$AR^{2/3}/D^{8/3}$
0.64	3.70918	0.53085	1.85459	0.96000	0.39475	0.23056
0.65	3.75098	0.54042	1.87549	0.95394	0.40676	0.23576
0.66	3.79305	0.54992	1.89653	0.94742	0.41897	0.24092
0.67	3.83543	0.55936	1.91771	0.94043	0.43140	0.24602
0.68	3.87813	0.56873	1.93906	0.93295	0.44405	0.25106
0.69	3.92119	0.57802	1.96059	0.92499	0.45693	0.25604
0.70	3.96463	0.58723	1.98231	0.91652	0.47005	0.26095
0.71	4.00848	0.59635	2.00424	0.90752	0.48342	0.26579
0.72	4.05279	0.60538	2.02639	0.89800	0.49705	0.27054
0.73	4.09758	0.61431	2.04879	0.88702	0.51097	0.27520
0.74	4.14290	0.62313	2.07145	0.87727	0.52518	0.27976
0.75	4.18879	0.63185	2.09440	0.86603	0.53971	0.28422
0.76	4.23529	0.64045	2.11765	0.85417	0.55457	0.28856
0.77	4.28247	0.64893	2.14123	0.84167	0.56981	0.29279
0.78	4.33036	0.65728	2.16518	0.82849	0.58544	0.29689
0.79	4.37905	0.66550	2.18953	0.81462	0.60151	0.30085
0.80	4.42859	0.67357	2.21430	0.80000	0.61806	0.30466
0.81	4.47908	0.68150	2.23954	0.78460	0.63514	0.30832
0.82	4.53059	0.68926	2.26529	0.76837	0.65282	0.31181
0.83	4.58323	0.69686	2.29162	0.75127	0.67116	0.31513
0.84	4.63712	0.70429	2.31856	0.73321	0.69025	0.31825
0.85	4.69239	0.71152	2.34619	0.71414	0.71022	0.32117
0.86	4.74920	0.71856	2.37460	0.69397	0.73119	0.32388
0.87	4.80773	0.72540	2.40387	0.67261	0.75333	0.32635
0.88	4.86822	0.73201	2.43411	0.64992	0.77687	0.32858
0.89	4.93092	0.73839	2.46546	0.62578	0.80208	0.33053
0.90	4.99618	0.74452	2.49809	0.60000	0.82936	0.33219
0.91	5.06441	0.75039	2.53221	0.57236	0.85919	0.33354
0.92	5.13616	0.75596	2.56808	0.54259	0.89231	0.33453
0.93	5.21213	0.76123	2.60607	0.51029	0.92974	0.33512
0.94	5.29332	0.76616	2.64666	0.47497	0.97307	0.33527
0.95	5.38113	0.77072	2.69057	0.43589	1.02483	0.33491
0.96	5.47775	0.77486	2.73888	0.39192	1.08953	0.33393
0.97	5.58685	0.77853	2.79343	0.34117	1.17605	0.33218
0.98	5.71560	0.78165	2.85780	0.28000	1.30599	0.32936
0.99	5.88252	0.78407	2.94126	0.19900	1.55635	0.32476
1.00	6.28319	0.78540	3.14159	0.00000		0.31169

*The notations 'E + a' represents 10^a and 'E - a' represents 10^{-a} . Thus for example

$$0.13523\text{E}+01 = 1.3523$$

$$0.47006\text{E}-01 = 0.047006$$

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ξ	ψ	ξ	ψ	ξ	ψ	ξ	ψ	ξ	ψ
0.100	0.0333042	0.330	0.2256807	0.560	0.5607910	0.790	1.0469124	1.020	1.6962526
0.105	0.0359281	0.335	0.2314360	0.565	0.5697107	0.795	1.0592476	1.025	1.7122746
0.110	0.0386272	0.340	0.2372580	0.570	0.5787019	0.800	1.0716601	1.030	1.7283798
0.115	0.0414006	0.345	0.2431469	0.575	0.5877645	0.805	1.0841500	1.035	1.7445682
0.120	0.0442474	0.350	0.2491026	0.580	0.5968989	0.810	1.0967174	1.040	1.7608400
0.125	0.0471671	0.355	0.2551252	0.585	0.6061050	0.815	1.1093625	1.045	1.7771953
0.130	0.0501588	0.360	0.2612149	0.590	0.6153829	0.820	1.1220854	1.050	1.7936343
0.135	0.0532222	0.365	0.2673716	0.595	0.6247330	0.825	1.1348861	1.055	1.8101570
0.140	0.0563565	0.370	0.2735954	0.600	0.6341551	0.830	1.1477649	1.060	1.8267635
0.145	0.0595615	0.375	0.2798865	0.605	0.6436496	0.835	1.1607219	1.065	1.8434541
0.150	0.0628365	0.380	0.2862449	0.610	0.6532164	0.840	1.1737572	1.070	1.8602288
0.155	0.0661812	0.385	0.2926706	0.615	0.6628558	0.845	1.1868709	1.075	1.8770877
0.160	0.0695953	0.390	0.2991638	0.620	0.6725678	0.850	1.2000631	1.080	1.8940310
0.165	0.0730784	0.395	0.3057246	0.625	0.6823525	0.855	1.2133341	1.085	1.9110589
0.170	0.0766302	0.400	0.3123531	0.630	0.6922102	0.860	1.2266838	1.090	1.9281713
0.175	0.0802504	0.405	0.3190493	0.635	0.7021409	0.865	1.2401125	1.095	1.9453685
0.180	0.0839387	0.410	0.3258133	0.640	0.7121448	0.870	1.2536203	1.100	1.9626506
0.185	0.0876950	0.415	0.3326452	0.645	0.7222220	0.875	1.2672072	1.105	1.9800176
0.190	0.0915190	0.420	0.3395452	0.650	0.7323725	0.880	1.2808735	1.110	1.9974698
0.195	0.0954105	0.425	0.3465132	0.655	0.7425966	0.885	1.2946192	1.115	2.0150072
0.200	0.0993694	0.430	0.3535495	0.660	0.7528944	0.890	1.3084445	1.120	2.0326299
0.205	0.1033955	0.435	0.3606541	0.665	0.7632659	0.895	1.3223496	1.125	2.0503382
0.210	0.1074887	0.440	0.3678272	0.670	0.7737114	0.900	1.3363344	1.130	2.0681321
0.215	0.1116488	0.445	0.3750688	0.675	0.7842309	0.905	1.3503992	1.135	2.0860117
0.220	0.1158757	0.450	0.3823789	0.680	0.7948246	0.910	1.3645441	1.140	2.1039771
0.225	0.1201694	0.455	0.3897579	0.685	0.8054926	0.915	1.3787693	1.145	2.1220286
0.230	0.1245297	0.460	0.3972056	0.690	0.8162350	0.920	1.3930747	1.150	2.1401661
0.235	0.1289566	0.465	0.4047224	0.695	0.8270520	0.925	1.4074607	1.155	2.1583899
0.240	0.1334500	0.470	0.4123082	0.700	0.8379437	0.930	1.4219272	1.160	2.1767000
0.245	0.13890098	0.475	0.4199631	0.705	0.8489102	0.935	1.4364745	1.165	2.1950965
0.250	0.1426361	0.480	0.4276873	0.710	0.8599516	0.940	1.4511026	1.170	2.2135797
0.255	0.1473287	0.485	0.4354810	0.715	0.8710681	0.945	1.4658118	1.175	2.2321496
0.260	0.1520877	0.490	0.4433441	0.720	0.8822598	0.950	1.4806020	1.180	2.2508063
0.265	0.1569130	0.495	0.4512768	0.725	0.8935269	0.955	1.4954734	1.185	2.2695499
0.270	0.1618046	0.500	0.4592793	0.730	0.9048694	0.960	1.5104263	1.190	2.2883806
0.275	0.1667625	0.505	0.4673517	0.735	0.9162875	0.965	1.5254606	1.195	2.3072986
0.280	0.1717868	0.510	0.4754940	0.740	0.9277813	0.970	1.5405765	1.200	2.3263038
0.285	0.1768773	0.515	0.4837063	0.745	0.9393510	0.975	1.5557742	1.205	2.3453965
0.290	0.1820342	0.520	0.4919889	0.750	0.9509966	0.980	1.5710537	1.210	2.3645767
0.295	0.1872575	0.525	0.5003418	0.755	0.9627183	0.985	1.5864153	1.215	2.3838447
0.300	0.1925471	0.530	0.5087651	0.760	0.9745163	0.990	1.6018590	1.220	2.4032004
0.305	0.1979031	0.535	0.5172590	0.765	0.9863907	0.995	1.6173849	1.225	2.4226440
0.310	0.2033256	0.540	0.5258236	0.770	0.9983415	1.000	1.6329932	1.230	2.4421757
0.315	0.2088145	0.545	0.5344589	0.775	1.0103690	1.005	1.6486840	1.235	2.4617956
0.320	0.2143700	0.550	0.5431652	0.780	1.0224732	1.010	1.6644574	1.240	2.4815037
0.325	0.2199920	0.555	0.5519425	0.785	1.0346543	1.015	1.6803135	1.245	2.5013003
0.330	0.2256807	0.560	0.5607910	0.790	1.0469124	1.020	1.6962526	1.250	2.5211853

In Table 2A. 2 $\xi = \frac{my_c}{B}$
 $\psi = \frac{\alpha Q^2 m^3}{gB^5}$

Uniform Flow

3

3.1 INTRODUCTION

A flow is said to be uniform if its properties remain constant with respect to distance. As mentioned earlier, the term uniform flow in open channels is understood to mean steady uniform flow. The depth of flow remains constant at all sections in a uniform flow (Fig. 3.1). Considering two Sections 1 and 2, the depths

$$\begin{aligned} y_1 &= y_2 = y_0 \\ A_1 &= A_2 = A_0 \end{aligned}$$

and hence

Since $Q = AV = \text{constant}$, it follows that in uniform flow $V_1 = V_2 = V$. Thus in a uniform flow, the depth of flow, area of cross-section and velocity of flow remain constant along the channel. It is obvious, therefore, that uniform flow is possible only in prismatic channels. The trace of the water surface and channel bottom slope are parallel in uniform flow (Fig. 3.1). Further, since $V = \text{constant}$, the energy line will be at a constant elevation above the water surface. As such, the slope of the energy line S_f , slope of the water surface S_w and bottom slope S_0 will all be equal to each other.

3.2 CHEZY EQUATION

By definition there is no acceleration in uniform flow. By applying the momentum equation to a control volume encompassing Sections 1 and 2, distance L apart, as shown in Fig. 3.1,

$$P_1 - W \sin \theta - F_f - P_2 = M_2 - M_1 \quad (3.1)$$

where P_1 and P_2 are the pressure forces and M_1 and M_2 are the momentum fluxes at Sections 1 and 2 respectively $W = \text{weight to fluid in the control volume}$ and $F_f = \text{shear force at the boundary}$.

Since the flow is uniform,

$$\begin{aligned} P_1 &= P_2 & \text{and} & & M_1 &= M_2 \\ W &= \gamma AL & \text{and} & & F_f &= \tau_0 PL \end{aligned}$$

Also,

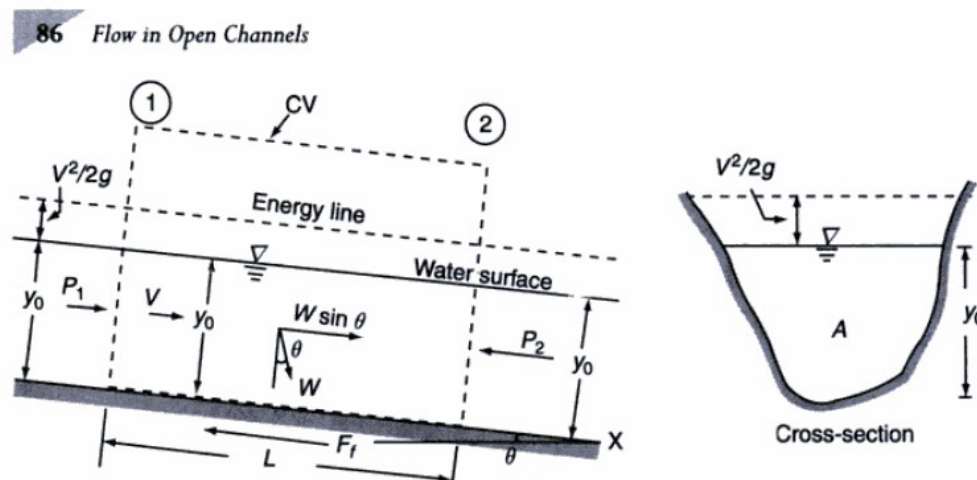


Fig. 3.1 Uniform flow

where τ_0 = average shear stress on the wetted perimeter of length P and γ = unit weight of water. Replacing $\sin \theta$ by S_0 (= bottom slope), Eq. 3.1 can be written as

$$\gamma ALS_0 = \tau_0 PL$$

or

$$\tau_0 = \gamma \frac{A}{P} S_0 = \gamma RS_0 \quad (3.2)$$

where $R = A/P$ is defined as the *hydraulic radius*. R is a length parameter accounting for the shape of the channel. It plays a very important role in developing flow equations which are common to all shapes of channels.

Expressing the average shear stress τ_0 as $\tau_0 = k\rho V^2$, where k = a coefficient which depends on the nature of the surface and flow parameters, Eq. 3.2 is written as

$$k\rho V^2 = \gamma RS_0$$

leading to

$$V = C \sqrt{RS_0} \quad (3.3)$$

where $C = \sqrt{\frac{\gamma}{\rho k}} = \frac{1}{\sqrt{k}}$ is a coefficient which depends on the nature of the surface and the

flow. Equation 3.3 is known as the *Chezy formula* after the French engineer Antoine Chezy, who is credited with developing this basic simple relationship in 1769. The dimensions of C are $[L^{1/2} T^{-1}]$ and it can be made dimensionless by dividing it by $\sqrt{g}$. The coefficient C is known as the Chezy coefficient.

3.3 DARCY-WEISBACH FRICTION FACTOR f

Incompressible, turbulent flow over plates, in pipes and ducts have been extensively studied in the fluid mechanics discipline. From the time of Prandtl (1875–1953) and

Von Karman (1881–1963) research by numerous eminent investigators has enabled considerable understanding of turbulent flow and associated useful practical applications. The basics of velocity distribution and shear resistance in a turbulent flow are available in any good text on fluid mechanics^{1,2}.

Only relevant information necessary for our study is summed up in this section.

Pipe Flow A surface can be termed hydraulically smooth, rough or in transition depending on the relative thickness of the roughness magnitude to the thickness of the laminar sub-layer. The classification is as follows:

$$\frac{\varepsilon_s \nu_*}{\nu} < 4 \text{ — hydraulically-smooth wall}$$

$$4 < \frac{\varepsilon_s \nu_*}{\nu} < 60 \text{ — transitional regime}$$

$$\frac{\varepsilon_s \nu_*}{\nu} > 60 \text{ — full rough flow}$$

where ε_s = equivalent sand grain roughness, $\nu_* = \sqrt{\tau_0 / \rho} = \sqrt{gRS_0}$ = shear velocity and

ν = kinematic viscosity.

For pipe flow, the Darcy–Weisbach equation is

$$h_f = f \frac{L}{D} \frac{V^2}{2g} \quad (3.4)$$

where h_f = head loss due to friction in a pipe of diameter D and length L ; f = Darcy–Weisbach friction factor. For smooth pipes, f is found to be a function of the Reynolds number $\left(\text{Re} = \frac{VD}{\nu} \right)$ only. For rough turbulent flows, f is a function of the relative roughness (ε_s/D) and type of roughness and is independent of the Reynolds number. In the transition regime, both the Reynolds number and relative roughness play important roles. The roughness magnitudes for commercial pipes are expressed as equivalent sand-grain roughness ε_s . The extensive experimental investigations of pipe flow have yielded the following generally accepted relations for the variation of f in various regimes of flow:

1. For smooth walls and $\text{Re} < 10^5$

$$f = \frac{0.316}{\text{Re}^{1/4}} \quad (\text{Blasius formula}) \quad (3.5)$$

2. For smooth walls and $\text{Re} > 10^5$

$$\frac{1}{\sqrt{f}} = 2.0 \log \text{Re} \sqrt{f} - 0.8 \quad (\text{Karman–Prandtl equation}) \quad (3.6)$$

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3. For rough boundaries and $Re > 10^5$

$$\frac{1}{\sqrt{f}} = -2 \log \frac{\epsilon_s}{D} + 1.14 \quad (\text{Karman-Prandtl equation}) \quad (3.7)$$

4. For the transition zone

$$\frac{1}{\sqrt{f}} + 2 \log \frac{\epsilon_s}{D} = 1.14 - 2 \log \left(1 + 9.35 \frac{D/\epsilon_s}{Re\sqrt{f}} \right) \quad (\text{Colebrook-White equation}) \quad (3.8)$$

It is usual to show the variation of f with Re and $\frac{\epsilon_s}{D}$ by a three-parameter graph known as the *Moody chart*.

Studies on non-circular conduits, such as rectangular, oval and triangular shapes have shown that by introducing the hydraulic radius R , the formulae developed for pipes are applicable for non-circular ducts also. Since for a circular shape $R = D/4$, by replacing D by $4R$, Eqs 3.5 to 3.8 can be used for any duct shape provided the conduit areas are close enough to the area of a circumscribing circle or semicircle.

Open Channels For purposes of flow resistance which essentially takes place in a thin layer adjacent to the wall, an open channel can be considered to be a conduit cut into two. The hydraulic radius would then be the appropriate length parameter and prediction of friction factor f can be done by using Eqs 3.5 to 3.8. It should be remembered that $Re = \frac{4RV}{\nu}$ and the relative roughness is $(\epsilon_s/4R)$.

Equation 3.4 can then be written for an open channel flow as

$$h_f = f \frac{L}{4R} \frac{V^2}{2g}$$

which on rearranging gives

$$V = \sqrt{\frac{8g}{f}} \sqrt{R} \cdot \sqrt{h_f/L} \quad (3.9)$$

Noting that for uniform flow in an open channel $h_f/L = \text{slope of the energy line} = S_f = S_0$, it may be seen that Eq. 3.9 is the same as Eq. 3.3 (Chezy formula) with

$$C = \sqrt{8g/f} \quad (3.10)$$

For convenience of use, Eq. 3.10 along with Eqs 3.5 to 3.8 can be used to prepare a modified Moody chart showing the variation of C with

$$\left(Re = \frac{4RV}{\nu} \right) \text{ and } \left(\frac{4R}{\epsilon_s} \right)$$



If f is to be calculated by using one of the Eqs 3.5 to 3.8, Eqs 3.6 to 3.8 are inconvenient to use as f is involved on both sides of the equations. Simplified empirical forms of Eqs 3.6 and 3.8, which are accurate enough for all practical purposes, are given by Jain³ as follows:

$$\frac{1}{\sqrt{f}} = 1.80 \log \text{Re} - 1.5146 \quad (\text{in lieu of Eq. (3.6)}) \quad (3.6a)$$

and
$$\frac{1}{\sqrt{f}} = 1.14 - 2.0 \log \left(\frac{\epsilon_s}{4R} + \frac{21.25}{\text{Re}^{0.9}} \right) \quad (\text{in lieu of Eq. (3.8)}) \quad (3.8a)$$

Equation (3.8a) is valid for $5000 \leq \text{Re} \leq 10^8$ and $10^{-6} < \frac{\epsilon_s}{4R} < 10^{-2}$.

These two equations are very useful for obtaining explicit solutions of many flow-resistance problems.

Generally, the open channels that are encountered in the field are very large in size and also in the magnitude of roughness elements. Consequently, high Reynolds numbers and large relative roughnesses are operative with the result that most of the open channels have rough turbulent-flow regimes. Due to paucity of reliable experimental or field data on channels covering a wide range of parameters, values of ϵ_s are not available to the same degree of confidence as for pipe materials. However, Table 3.1 can be used to estimate the values of ϵ_s for some common open channel surfaces.

Table 3.1 Values of ϵ_s for some Common Channel Surfaces

Sl. No.	Surface	Equivalent Roughness ϵ_s in mm
1	Glass	3×10^{-4}
2	Very smooth concrete surface	0.15–0.30
3	Glazed sewer pipe	0.60
4	Gunit (smooth)	0.50–1.5
5	Rough concrete	3.0–4.5
6	Earth channels (straight, uniform)	3.0
7	Rubble masonry	6.0
8	Untreated gunit	3.0–10.0

3.4 MANNING'S FORMULA

A resistance formula proposed by Robert Manning, and Irish engineer, for uniform flow in open channels, is

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \quad (3.11)$$

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where n = a roughness coefficient known as Manning's n . This coefficient is essentially a function of the nature of boundary surface. It may be noted that the dimensions of n are $[L^{-1/3} T]$. Equation 3.11 is popularly known as the Manning's formula. Owing to its simplicity and acceptable degree of accuracy in a variety of practical applications, the Manning's formula is probably the most widely used uniform flow formula in the world.

Comparing Eq. 3.11 with the Chezy formula, Eq. 3.3, we have

$$C = \frac{1}{n} R^{1/6} \quad (3.12)$$

From Eq. 3.10,

$$C = \sqrt{\frac{8g}{f}} = \frac{1}{n} R^{1/6}$$

i.e.

$$f = \left(\frac{n^2}{R^{1/3}} \right) (8g) \quad (3.13)$$

Since Eq. 3.13 does not contain velocity term (and hence the Reynolds number), we can compare Eq. 3.13 with Eq. 3.7, i.e., the Prandtl-Karman relationship for rough turbulent flow. If Eq. 3.7 is plotted as f vs. $\frac{4R}{\epsilon_s}$ on a log-log paper, a smooth curve that can be approximated to a straight line with a slope of $\left(-\frac{1}{3}\right)$ is obtained (Fig 3.2). From this the term f can be expressed as

$$f \propto \left(\frac{4R}{\epsilon_s} \right)^{-1/3} \quad \text{or} \quad f \propto \left(\frac{\epsilon_s}{R} \right)^{1/3}$$

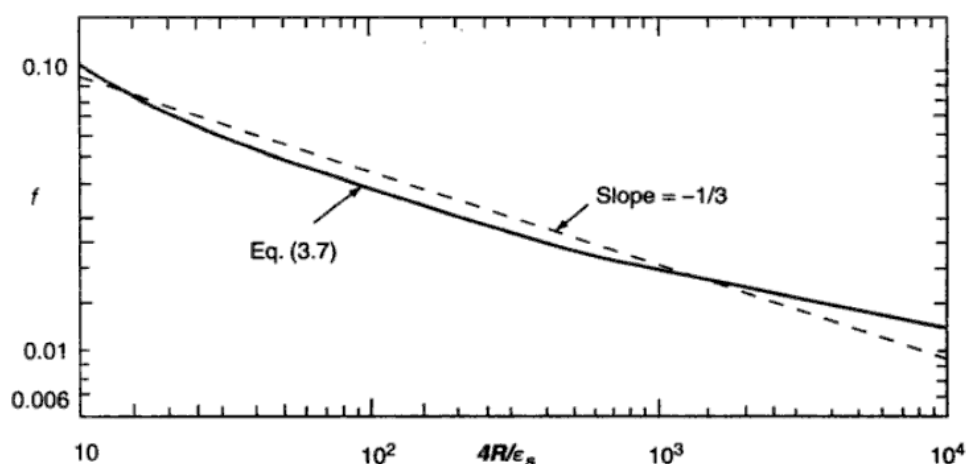


Fig. 3.2 Variation of f in fully rough flow



Since from Eq. 3.13, $f \propto \frac{n^2}{R^{1/3}}$, it follows that $n \propto \varepsilon_s^{1/6}$. Conversely, if $n \propto \varepsilon_s^{1/6}$, the Manning's formula and Darcy–Weisbach formula both represent rough turbulent flow $\left(\frac{\varepsilon_s v_*}{\nu} > 60 \right)$.

3.5 OTHER RESISTANCE FORMULAE

Several forms of expressions for the Chezy coefficient C have been proposed by different investigators in the past. Many of these are archaic and are of historic interest only. A few selected ones are listed below:

1. Pavlovski formula

$$C = \frac{1}{n} R^x \quad (3.14)$$

in which $x = 2.5 \sqrt{n} - 0.13 - 0.75 \sqrt{R} (\sqrt{n} - 0.10)$ and n = Manning's coefficient. This formula appears to be in use in Russia.

2. Ganguillet and Kutter Formula

$$C = \frac{23 + \frac{1}{n} + \frac{0.00155}{S_0}}{1 + \left[23 + \frac{0.00155}{S_0} \right] \frac{n}{\sqrt{R}}} \quad (3.15)$$

in which n = Manning's coefficient.

3. Bazin's formula

$$C = \frac{87.0}{1 + M/R} \quad (3.16)$$

in which M = a coefficient dependent on the surface roughness.

3.6 VELOCITY DISTRIBUTION

(a) Wide Channels

(i) *Velocity-defect Law* In channels with large aspect ratio B/y_0 , as for example in rivers and very large canals, the flow can be considered to be essentially two dimensional. The fully developed velocity distributions are similar to the logarithmic

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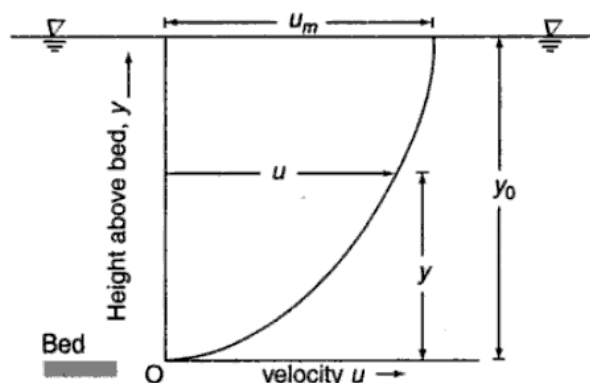


Fig. 3.3 Velocity profile in a wide open channel

form of velocity-defect law found in turbulent flow in pipes. The maximum velocity u_m occurs essentially at the water surface, (Fig. 3.3). The velocity u at a height y above the bed in a channel having uniform flow at a depth y_0 is given by the velocity-defect law for $y/y_0 > 0.15$ as

$$\begin{aligned} \frac{u_m - u}{u_*} &= \frac{1}{k} \ln \frac{y}{y_0} \\ &= -\frac{2.3}{k} \log_{10} (y/y_0) \end{aligned} \quad (3.17)$$

where u_* = shear velocity = $\sqrt{\tau_0 / \rho} = \sqrt{gRS_0}$, R = hydraulic radius, S_0 = longitudinal slope, and k = Karman constant = 0.41 for open channel flows⁵.

This equation is applicable to both rough and smooth boundaries alike. Assuming the velocity distribution of Eq. 3.17 is applicable to the entire depth y_0 , the velocity u can be expressed in terms of the average velocity

$$\begin{aligned} V &= \frac{1}{y_0} \int_0^{y_0} u \, dy \quad \text{as} \\ u &= V + \frac{u_*}{k} \left(1 + \ln \frac{y}{y_0} \right) \end{aligned} \quad (3.18)$$

From Eq. 3.18, it follows that

$$V = u_m - \frac{u_*}{k} \quad (3.19)$$

(ii) *Law of the Wall* For smooth boundaries, the law of the wall as

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{yu_*}{\nu} + A_s \quad (3.20)$$

is found applicable in the inner wall region ($y/y_0 < 0.20$). The values of the constants are found to be $k = 0.41$ and $A_s = 5.29$ regardless of the Froude number and Reynolds number of the flow⁵. Further, there is an overlap zone between the law of the wall region and the velocity-defect law region.

For completely rough turbulent flows, the velocity distribution in the wall region ($y/y_0 < 0.20$) is given by

$$\frac{u}{u_*} = \frac{1}{k} \ln \frac{y}{\varepsilon_s} + A_r \quad (3.21)$$

where ε_s = equivalent sand grain roughness. It has been found that k is a universal constant irrespective of the roughness size⁵. Values of $k = 0.41$ and $A_r = 8.5$ are appropriate.

For further details of the velocity distributions Ref. [5] can be consulted.

(b) Channels with Small Aspect Ratio In channels which are not wide enough to have two dimensional flow, the resistance of the sides will be significant to alter the two-dimensional nature of the velocity distribution given by Eq. 3.17. The most important feature of the velocity distributions in such channels is the occurrence of *velocity-dip*, where the maximum velocity occurs not at the free surface but rather some distance below it, (Fig. 3.4) Various investigations have inferred the secondary currents as responsible for this velocity-dip phenomenon⁴. The critical ratio of B/y_0 above which the velocity-dip becomes insignificant has been found to be about 5.0. Based on this the channels with $B/y_0 \leq 5$ can be classified as *narrow channels*.

Typical velocity distributions in rectangular channels with $B/y_0 = 1.0$ and 6.0 are shown in Figures 3.5(a) and (b) respectively.

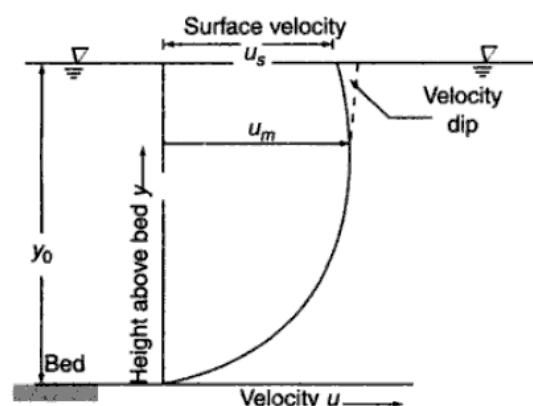


Fig. 3.4 Velocity profile in a narrow channel

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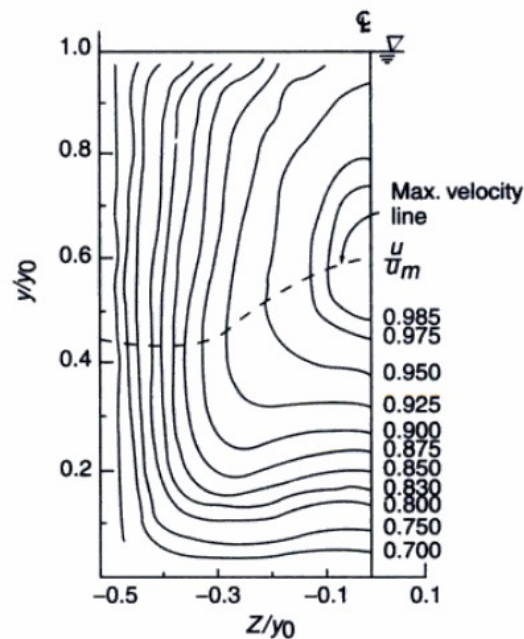


Fig. 3.5(a) Typical velocity distribution in a narrow channel, $B/y_0 = 1.0$. (Ref. 4)

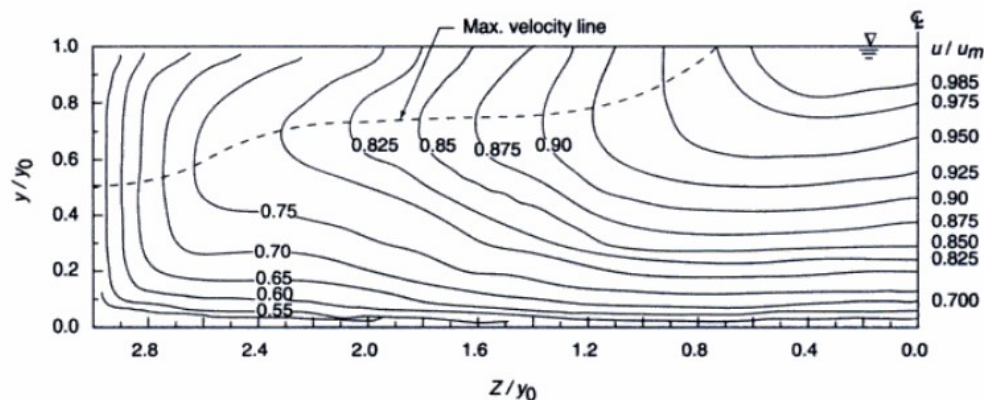


Fig. 3.5(b) Typical velocity distribution in a rectangular channel with $B/y_0 = 6.0$. (Ref. 47)

3.7 SHEAR STRESS DISTRIBUTION

The average shear stress τ_0 on the boundary of a channel is, by Eq. 3.2, given as $\tau_0 = \gamma R S_0$. However, this shear stress is not uniformly distributed over the boundary. It is zero at the intersection of the water surface with the boundary and also at the corners in the boundary. As such, the boundary shear stress τ_0 will have certain local maxima on the side as well as on the bed. The turbulence of the flow and the presence of secondary cur-

rents in the channel also contribute to the non-uniformity of the shear stress distribution. A knowledge of the shear stress distribution in a channel is of interest not only in the understanding of the mechanics of flow but also in certain problems involving sediment transport and design of stable channels in non-cohesive material (Chapter 11).

Preston tube⁵ is a very convenient device for the boundary shear stress measurements in a laboratory channel. Distributions of boundary shear stress by using Preston tube in rectangular, trapezoidal and compound channels have been reported^{6,7}. Isaacs and Macintosh⁸ report the use of a modified Preston tube to measure shear stress in open channels.

Lane⁹ obtained the shear stress distributions on the sides and bed of trapezoidal and rectangular channels by the use of membrane analogy. A typical distribution of the boundary shear stress on the side (τ_s) and bed (τ_b) in a trapezoidal channel of $B/y_0 = 4.0$ and side slope $m = 1.5$ obtained by Lane is shown in Fig 3.6. The variation of the maximum shear stress on the bed τ_{bm} and on the sides τ_{sm} in rectangular and trapezoidal channels is shown in Fig. 3.7. It is noted from the figure that for trapezoidal sections approximately $\tau_{sm} \approx 0.76 \gamma y_0 S_0$ and $\tau_{bm} \approx \gamma y_0 S_0$ when $B/y_0 \gtrsim 6.0$.

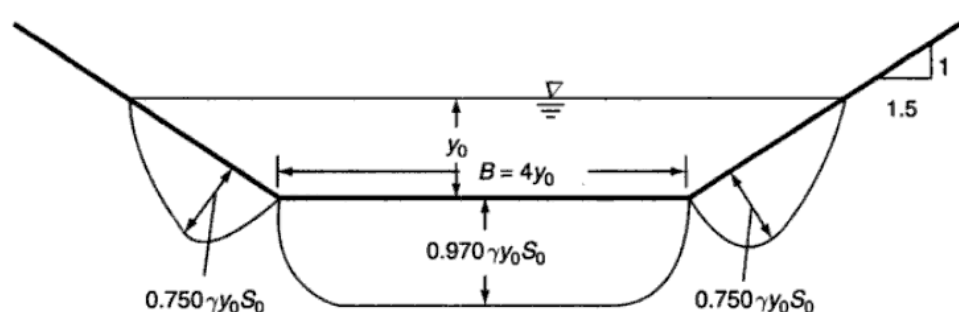


Fig. 3.6 Variation of boundary shear stress in a trapezoidal channel with $B/y_0 = 4$ and $m = 1.5$

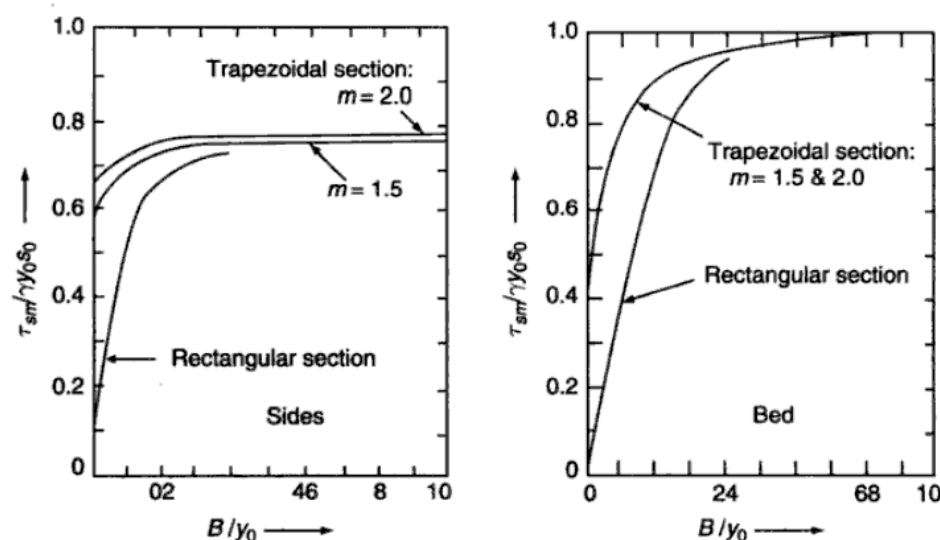


Fig. 3.7 Variation of maximum shear stress on bed and sides of smooth channels

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Since a majority of the open channel flow are in the rough turbulent range, the Manning's formula (Eq. 3.11) is the most convenient one for practical use. Since, it is simple in form and is also backed by considerable amount of experience, it is the most preferred choice of hydraulic engineers. However, it has a limitation in that it cannot adequately represent the resistance in situations where the Reynolds number effect is predominant and this must be borne in mind. In the book, the Manning's formula is used as the resistance equation.

The Darcy-Weisbach coefficient f used with the Chezy formula is also an equally effective way of representing the resistance in uniform flow. However, field engineers generally do not prefer this approach, partly because of the inadequate information to assist in the estimation of ϵ_s and partly because it is not sufficiently backed by experimental or field observational data. It should be realized that for open channel flows with hydrodynamically smooth boundaries, it is perhaps the only approach available to estimate the resistance.

3.9 MANNING'S ROUGHNESS COEFFICIENT n

In the Manning's formula, all the terms except n are capable of direct measurement. The roughness coefficient, being a parameter representing the integrated effects of the channel cross-sectional resistance, is to be estimated. The selection of a value for n is subjective, based on one's own experience and engineering judgement. However, a few aids are available which reduce to a certain extent the subjectiveness in the selection of an appropriate value of n for a given channel. These include:

1. Photographs of selected typical reaches of canals, their description and measured values of $n^{10,11}$. These act as type values and by comparing the channel under question with a figure and description set that resembles it most, one can estimate the value of n fairly well. Movies, stereoscopic colour photographs and video recordings of selected typical reaches are other possible effective aids under the category.
2. A comprehensive list of various types of channels, their descriptions with the associated range of values of n . Some typical values of n for various normally encountered channel surfaces prepared from information gathered from various sources^{10,11,12,13} are presented in Table 3.2.

Estimation of correct n -value of natural channels is of utmost importance in practical problems associated with backwater computations, flood flow estimation, routing and management. The photographs of man-made and natural channels with corresponding values of n given by Chow¹⁰, Barnes¹¹ and Arcemont and Schnieder¹⁴ are very useful in obtaining a first estimate of roughness coefficient in such situations.

Cowan¹⁵ has developed a procedure to estimate the value of roughness factor n of natural channels in a systematic way by giving weightages to various important factors that affect the roughness coefficient. According to Cowan.

$$n = (n_b + n_1 + n_2 + n_3 + n_4) m$$

Table 3.2 Values of Roughness Coefficient n

Sl. No.	Surface Characteristics	Range of n
(a)	<i>Lined channels with straight alignment</i>	
1	Concrete (a) formed, no finish	0.013–0.017
	(b) Trowel finish	0.011–0.015
	(c) Float finish	0.013–0.015
	(d) Gunite, good section	0.016–0.019
	(e) Gunite, wavy section	0.018–0.022
2	Concrete bottom, float finish, sides as indicated	
	(a) Dressed stone in mortar	0.015–0.017
	(b) Random stone in mortar	0.017–0.020
	(c) Cement rubble masonry	0.020–0.025
	(d) Cement-rubble masonry, plastered	0.016–0.020
	(e) Dry rubble (rip-rap)	0.020–0.030
3	Tile	0.016–0.018
4	Brick	0.014–0.017
5	Sewers (concrete, A.C., vitrified-clay pipes)	0.012–0.015
6	Asphalt (i) Smooth	0.013
	(ii) Rough	0.016
7	Concrete lined, excavated rock	
	(i) good section	0.017–0.020
	(ii) irregular section	0.022–0.027
8	Laboratory flumes-smooth metal bed and glass or perspex sides	0.009–0.010
(b)	<i>Unlined, non-erodible channels</i>	
1	Earth, straight and uniform	
	(i) clean, recently completed	0.016–0.020
	(ii) clean, after weathering	0.018–0.025
	(iii) gravel, uniform section, clean	0.022–0.030
	(iv) with short grass, few weeds	0.022–0.033
2	Channels with weeds and brush, uncut	
	(i) dense weeds, high as flow depth	0.05–0.12
	(ii) clean bottom, brush on sides	0.04–0.08
	(iii) dense weeds or aquatic plants in deep channels	0.03–0.035
	(iv) grass, some weeds	0.025–0.033
3	Rock	0.025–0.045
(c)	<i>Natural channels</i>	
1	Smooth natural earth channel, free from growth, little curvature	0.020
2	Earth channels, considerably covered with small growth	0.035
3	Mountain streams in clean loose cobbles, rivers with variable section with some vegetation on the banks	0.04–0.05
4	Rivers with fairly straight alignment, obstructed by small trees, very little under brush	0.06–0.075
5	Rivers with irregular alignment and cross-section, covered with growth of virgin timber and occasional patches of bushes and small trees	0.125

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Where n_b = a base value of n for a straight uniform smooth channel in natural material

n_1 = correction for surface irregularities

n_2 = correction for variation in shape and size of the cross section

n_3 = correction for obstructions

n_4 = correction for vegetation and flow conditions

m = correction for meandering of the channel

Values of the n_b and the other five correction factors are given in Chow¹⁰ and in Ref. (14).

Example 3.1

A 2.0-m wide rectangular channel carries water at 20°C at a depth of 0.5 m. The channel is laid on a slope of 0.0004. Find the hydrodynamic nature of the surface if the channel is made of (a) very smooth concrete and (b) rough concrete.

Solution Hydraulic radius $R = \frac{2 \times 0.5}{(2 + 2 \times 0.5)} = 0.333 \text{ m}$

$$\tau_0 = \gamma R S_0 = (9.81 \times 10^3) \times 0.333 \times 0.0004 = 1.308 \text{ N/m}^2$$

$$v_* = \text{shear velocity} = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\left(\frac{1.308}{10^3}\right)} = 0.03617 \text{ m/s}$$

(a) For a smooth concrete surface

From Table 3.1 $\varepsilon_s = 0.25 \text{ mm} = 0.00025 \text{ m}$

ν at 20°C = $10^{-6} \text{ m}^2/\text{s}$

$$\frac{\varepsilon_s v_*}{\nu} = \frac{0.00025 \times 0.03617}{10^{-6}} = 9.04$$

Since this value is slightly greater than 4.0, the boundary is hydrodynamically in the early transition from smooth to rough surface

(b) For a rough concrete surface

From Table 3.1, $\varepsilon_s = 3.5 \text{ mm} = 0.0035$

$$\frac{\varepsilon_s v_*}{\nu} = 126.6$$

Since this value is greater than 60, the boundary is hydrodynamically rough.

Example 3.2

For the two cases in Example 3.1, estimate the discharge in the channel using (i) the Chezy formula with Dancy-Weisbach f , and (ii) the, Manning's formula.

Solution Case (a): Smooth concrete channel

$$(i) \quad \varepsilon_s = 0.25 \text{ mm and } \frac{\varepsilon_s}{4R} = \frac{0.25}{4 \times 0.33 \times 10^3} = 1.894 \times 10^{-4}$$



$$\text{i.e.} \quad \sum P_i f_i = P f$$

$$\text{or} \quad f = \frac{\sum P_i f_i}{P} \quad (3.27)$$

Table 3.3 lists some of the equations proposed for estimation of equivalent roughness. This list is extracted from Ref. 16 which contains a list of 17 equations for composite roughness calculation.

Table 3.3 Equations for Equivalent Roughness Coefficient (Ref. 10, 16)

Sl. No	Investigator	n_e	Concept
1	Horton (1933); Einstein (1934)	$= \left[\frac{1}{P} \sum (n_i^{3/2} P_i) \right]^{2/3}$	Mean Velocity is constant in all subareas.
2	Pavlovskii (1931) Muhlhofer (1933) Einstein and Banks (1950)	$= \left[\frac{1}{P} \sum (n_i^2 P_i) \right]^{1/2}$	Total resistance force F is sum of subarea resistance forces, $\sum F_i$
3	Lotter (1932)	$= \frac{P R^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}}$	Total discharge is sum of subarea discharges
4	Felkel (1960)	$= \frac{P}{\sum \frac{P_i}{n_i}}$	Total discharge is sum of subarea discharges
5	Krishnamurthy and Christensen (1972)	$= \exp \left[\frac{\sum P_i h_i^{3/2} \ln n_i}{\sum P_i h_i^{3/2}} \right]$	Logarithmic velocity distribution over depth h for wide channel
6	Yen (1991)	$= \frac{\sum (n_i P_i)}{P}$	Total shear velocity is weighted sum of subarea shear velocity

Example 3.3 An earthen trapezoidal channel ($n = 0.025$) has a bottom width of 5.0 m, side slopes of 1.5 horizontal: 1 vertical and a uniform flow depth of 1.1 m. In an economic study to remedy excessive seepage from the canal two proposals, viz (a) to line the sides only, and (b) to line the bed only are considered. If the lining is of smooth concrete ($n = 0.012$), determine the equivalent roughness in the above two cases by using (i) Horton's formula, and by (ii) Pavlovskii formula.

Solution Case (a) Lining of the sides only

Here for the bed: $n_1 = 0.025$, and $P_1 = 5.0$ m.

For the sides: $n_2 = n_3 = 0.012$, and $P_2 = P_3 = 1.10 \times \sqrt{1 + (1.5)^2} = 1.983$ m

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(i) Equivalent roughness n_e by Horton's formula: $n_e = \left[\frac{1}{P} \sum (n_i^{3/2} P_i) \right]^{2/3}$

$$n_e = \frac{[5.0 \times (0.025)^{3/2} + 1.983 \times (0.012)^{3/2} + 1.983 \times (0.012)^{3/2}]^{2/3}}{[5.0 + 1.983 + 1.983]^{2/3}} = \frac{0.085448}{4.31584} = 0.0198$$

(ii) Equivalent roughness n_e by Pavlovskii formula: $n_e = \left[\frac{1}{P} \sum (n_i^2 P_i) \right]^{1/2}$

$$n_e = \frac{[5.0 \times (0.025)^2 + 1.983 \times (0.012)^2 + 1.983 \times (0.012)^2]^{1/2}}{[5.0 + 1.983 + 1.983]^{1/2}} = \frac{0.060796}{2.99433} = 0.0203$$

Case (b) Lining of the bed only

Here for the bed: $n_1 = 0.012$ and $P_1 = 5.0$ m.

For the sides: $n_2 = n_3 = 0.025$, and $P_2 = P_3 = 1.10 \times \sqrt{1 + 1.5^2} = 1.983$ m

(i) Equivalent roughness n_e by Horton's formula: $n_e = \left[\frac{1}{P} \sum (n_i^{3/2} P_i) \right]^{2/3}$

$$n_e = \frac{[5.0 \times (0.012)^{3/2} + 1.983 \times (0.025)^{3/2} + 1.983 \times (0.025)^{3/2}]^{2/3}}{[5.0 + 1.983 + 1.983]^{2/3}} = \frac{0.079107}{4.31584} = 0.01833$$

(ii) Equivalent roughness n_e by Pavlovskii formula: $n_e = \left[\frac{1}{P} \sum (n_i^2 P_i) \right]^{1/2}$

$$n_e = \frac{[5.0 \times (0.012)^2 + 1.983 \times (0.025)^2 + 1.983 \times (0.025)^2]^{1/2}}{[5.0 + 1.983 + 1.983]^{1/2}} = \frac{0.05656}{2.99433} = 0.01889$$

3.11 UNIFORM FLOW COMPUTATIONS

The Manning's formula (Eq. 3.11) and the continuity equation, $Q = AV$ form the basic equations for uniform-flow computations. The discharge Q is then given by

$$Q = \frac{1}{n} AR^{2/3} S_0^{1/2} \quad (3.28)$$

$$= K \sqrt{S_0} \quad (3.28a)$$

where, $K = \frac{1}{n} AR^{2/3}$ is called the *conveyance* of the channel and expresses the discharge capacity of the channel per unit longitudinal slope. The term $nK = AR^{2/3}$ is sometimes called the *section factor for uniform-flow computations*.