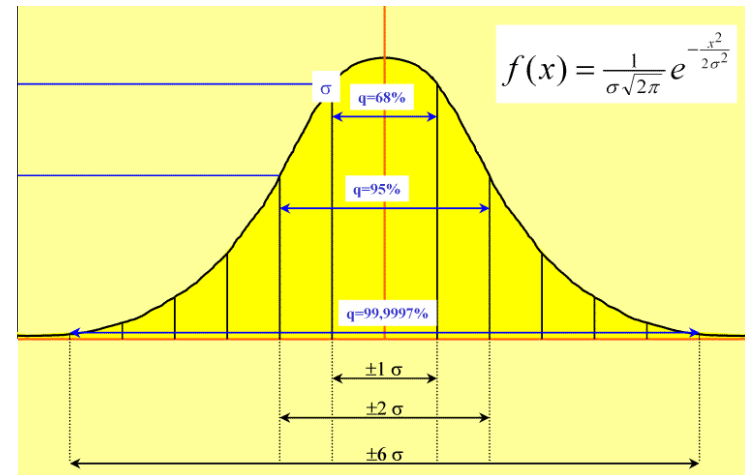
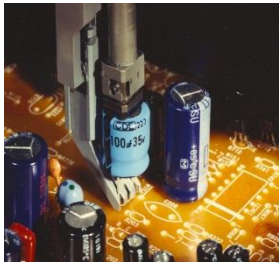


CH-3: CONTROL CHARTS FOR VARIABLES

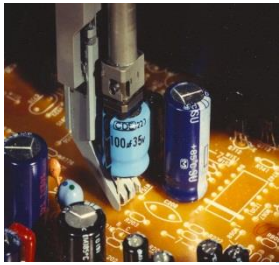




Learning Objectives

When you have completed this chapter you should:

- ☐ Know the three categories of variation and their sources.
- ☐ Understand the concept of the control chart method.
- ☐ Know the purpose of variable control charts.
- ☐ Know how to select the quality characteristics
- ☐ Be able to calculate the central value, and control limits for $\bar{X}$ bar and R chart.
- ☐ Know the difference between individual measurements and averages; control limits and specifications



variation

The variation concept is a law of nature in that no two natural items in any category are the same.

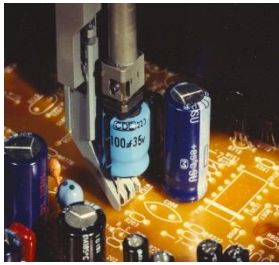
- ❑ The variation may be quite large and easily noticeable
- ❑ The variation may be very small. It may appear that items are identical; however, precision instruments will show difference
- ❑ The ability to measure variation is necessary before it can be controlled



Variation

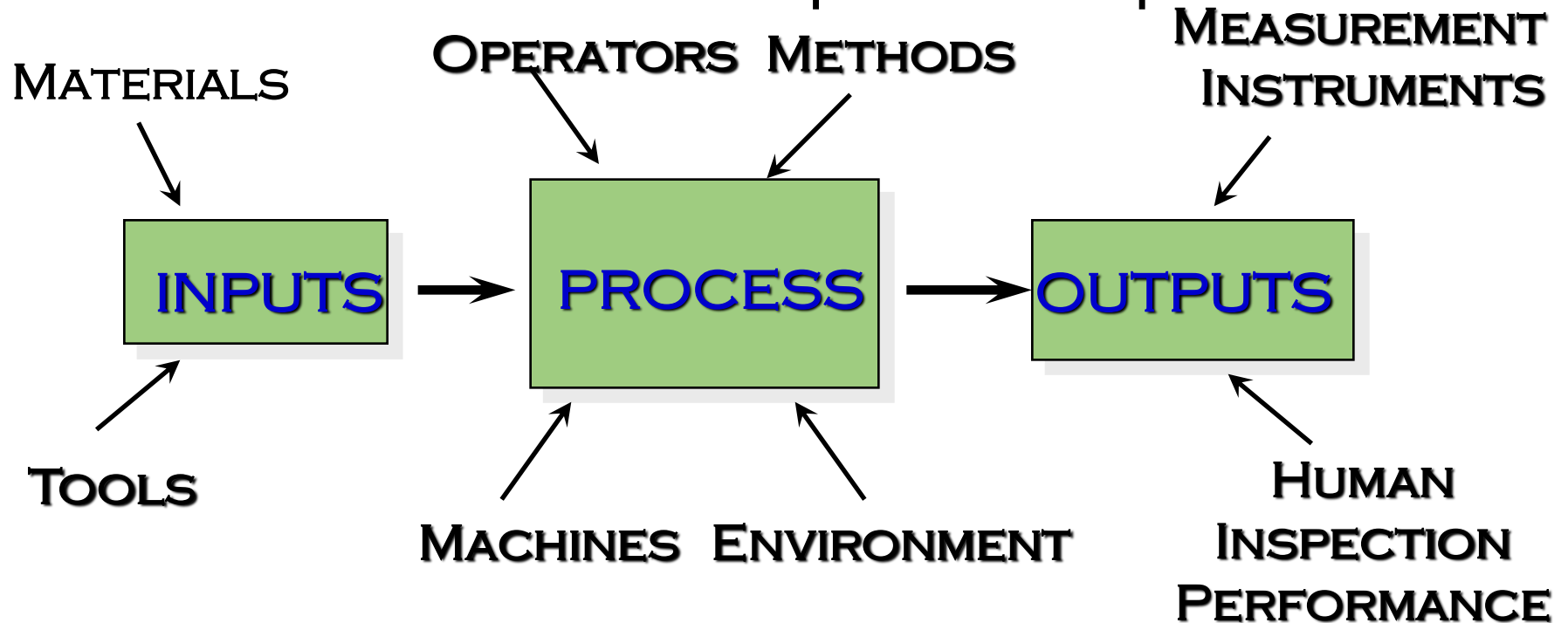
There are three categories of variation in piece part production:

1. Within-piece variation: Surface
2. Piece-to-piece variation: Among pieces produced at the same time
3. Time-to-time variation: Difference in product produced at different times of the day



Variation

Sources of Variation in production processes:





Variation

Sources of variation are:

1. Equipment:
 1. Tool wear
 2. Machine vibration
 3. Electrical fluctuations etc.
2. Material
 1. Tensile strength
 2. Ductility
 3. Thickness
 4. Porosity etc.



Variation

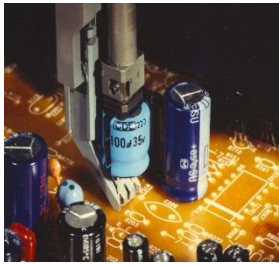
Sources of variation are:

3. Environment
 1. Temperature
 2. Light
 3. Radiation
 4. Humidity etc.
4. Operator
 1. Personal problem
 2. Physical problem etc.



Variation

There is also a reported variation which is due to the inspection activity. Variation due to inspection should account for one tenth of the four other sources of variation.

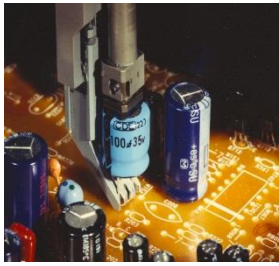


Variation

Variation may be due to **chance causes** (random causes) or **assignable causes**.

When only **chance causes** are present, then the process is said to be in a state of statistical control. The process is stable and predictable.

When **Assignable causes** are present, then the process is said to be in a state statistically out of control. The process is unstable and unpredictable overtime.



Quality Characteristic

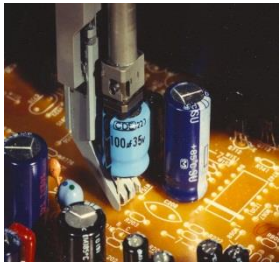
The Quality characteristic must be measurable. Quality characteristics can be expressed in terms of, for example, the following basic units:

1. Length
2. Mass
3. Time
4. Electrical current
5. Temperature
6. Substance
7. Luminosity

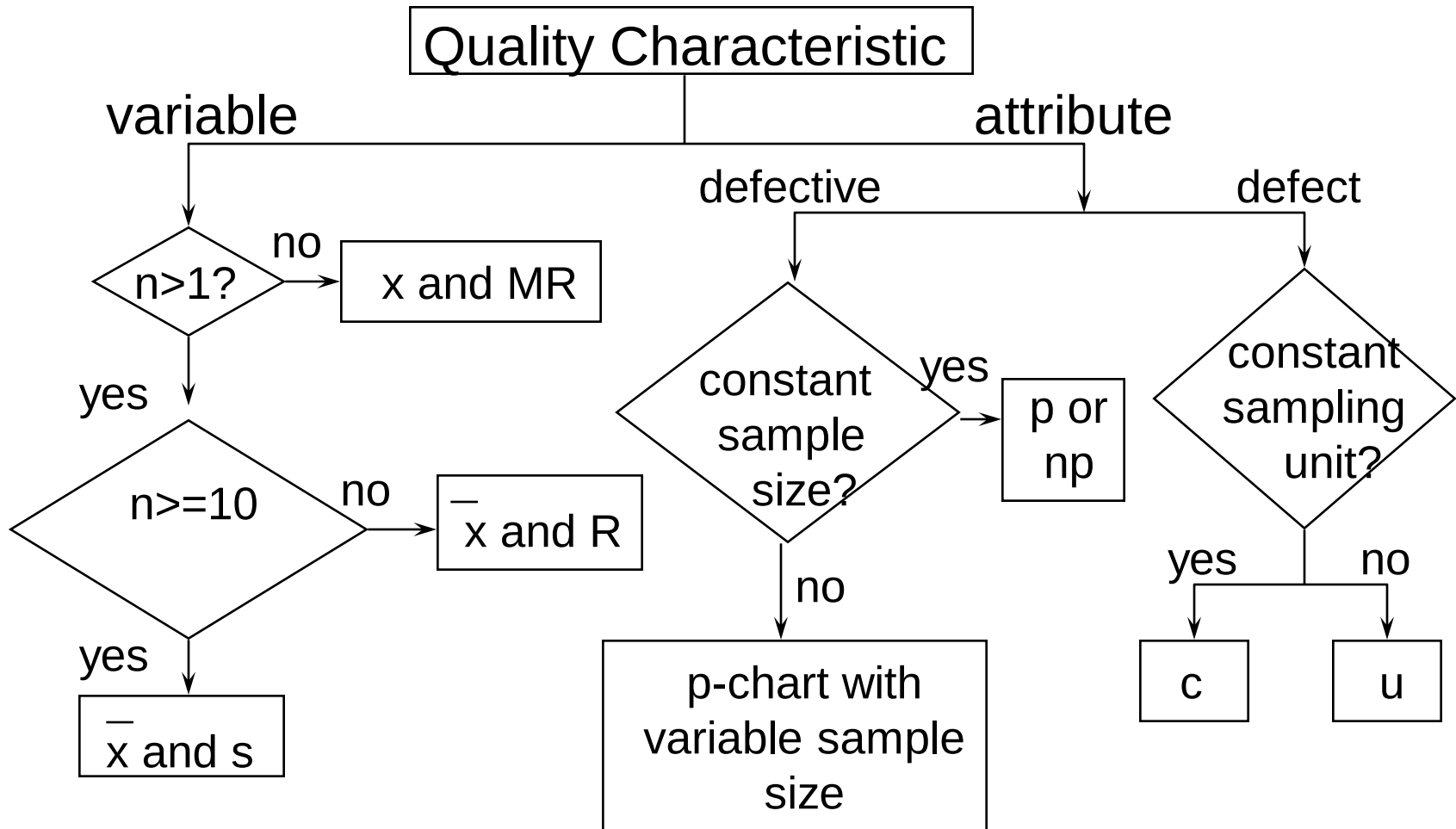


Control Charts

- ❑ Variable data
 - ❑ $\bar{x}$ -bar and R-charts
 - ❑ $\bar{x}$ -bar and s-charts
 - ❑ Charts for individuals (x-charts)
- ❑ Attribute data
 - ❑ For “defectives” (p-chart, np-chart)
 - ❑ For “defects” (c-chart, u-chart)



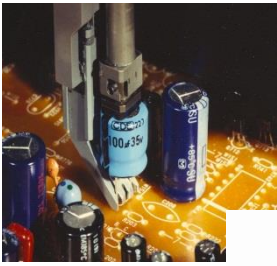
Control Chart Selection





Control Charts for Variables

The control chart for variables is a means of visualizing the variations that occur in the central tendency and the mean of a set of observations. It shows whether or not a process is in a stable state.



Control Charts

$\bar{X}$ Chart

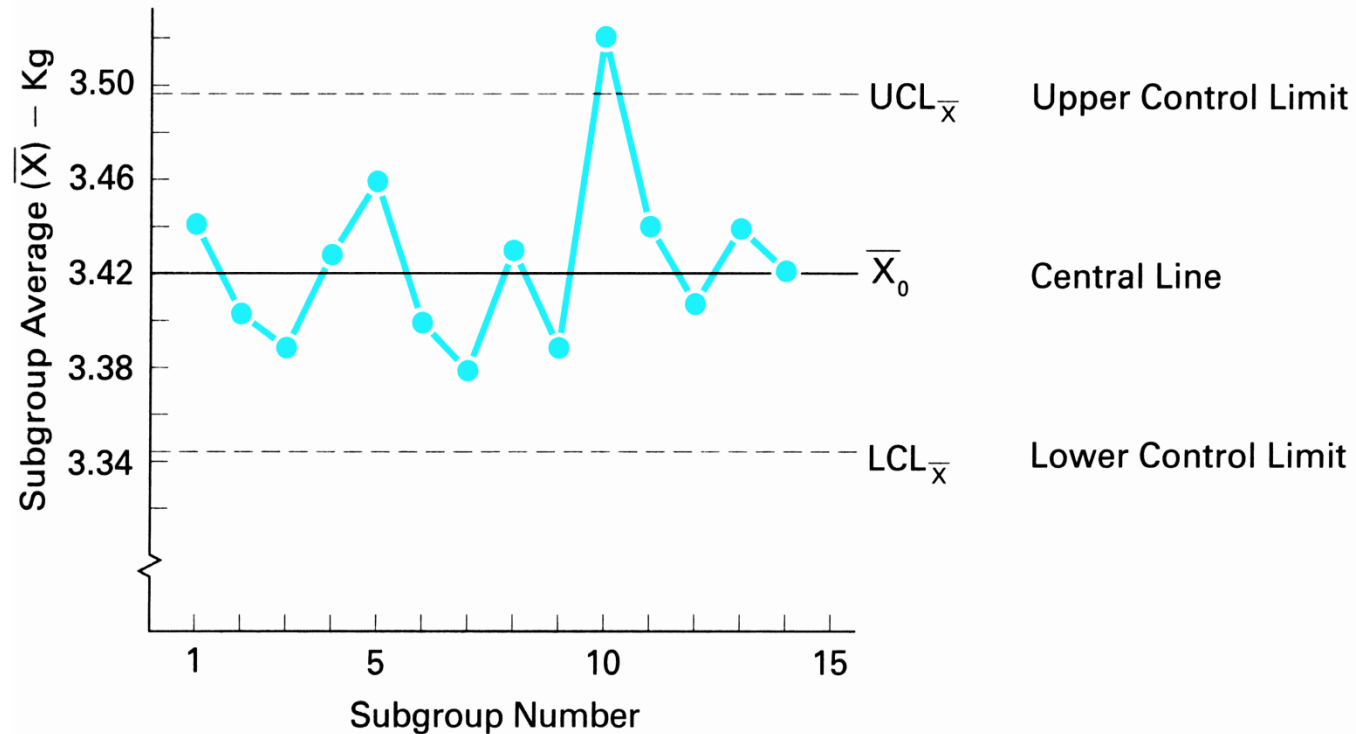


Figure 5-1 Example of a control chart



Control Charts

$\bar{X}$ AND R CHART

Work Center Number 365-2
 Quality Characteristic Durometer Date 3/6

Time	8:30 AM	9:30 AM	10:40 AM	11:50 AM	1:30 PM									
Subgroup	1	2	3	4	5	6	7	8	9	10	11	12	13	14
X_1	55	51	48	45	53									
X_2	52	52	49	43	50									
X_3	51	57	50	45	48									
X_4	53	50	49	43	50									
Sum	211	210	196	176	201									
$\bar{X}$	52.8	52.5	49	44	50.2									
R	4	7	2	2	5									

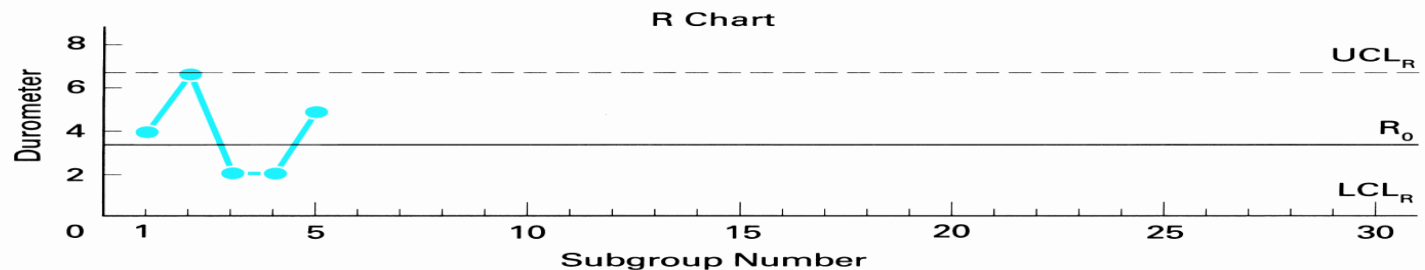
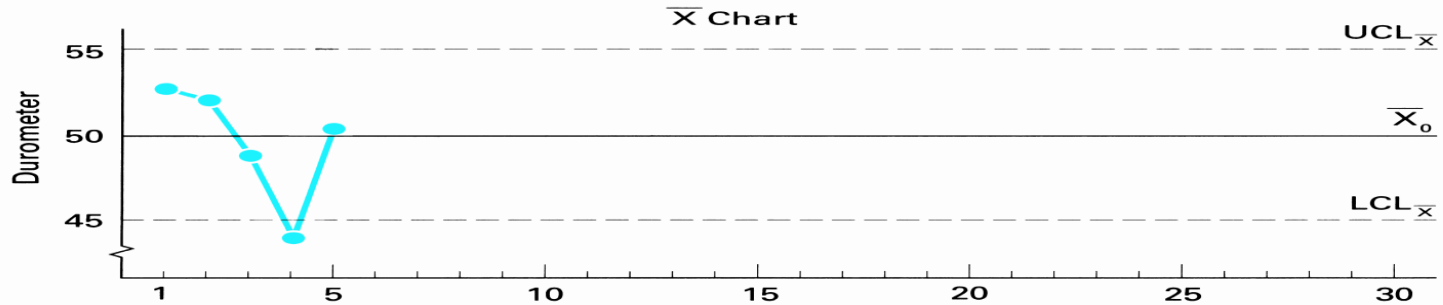
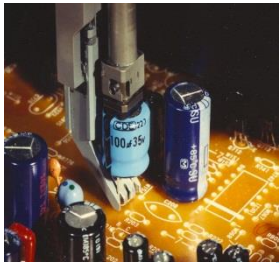


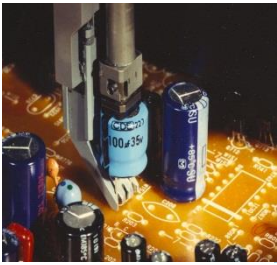
Figure 5-1 Example of a method of reporting inspection results



Variable Control Charts

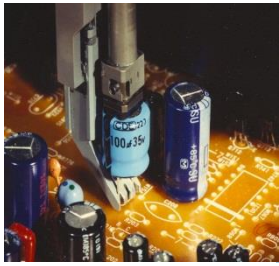
The objectives of the variable control charts are:

1. For quality improvement
2. To determine the process capability
3. For decisions regarding product specifications
4. For current decisions on the production process
5. For current decisions on recently produced items



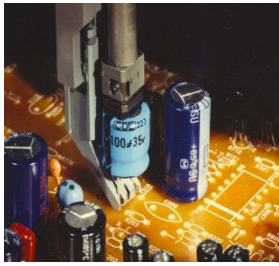
Steps in Creating Control Charts

1. Take representative sample from output of a process over a long period of time, e.g. 10 units every hour for 24 hours.
2. Compute means and ranges for the variables and calculate the control limits
3. Draw control limits on the control chart
4. Plot a chart for the means and another for the mean of ranges on the control chart
5. Determine state of process (in or out of control)
6. Investigate possible reasons for out of control events and take corrective action
7. Continue sampling of process output and reset the control limits when necessary



Control Charts for Variables

- ☑ For variables that have continuous dimensions
 - ☑ Weight, speed, length, strength, etc.
- ☑ $\bar{x}$ -charts are to control the central tendency of the process
- ☑ R-charts are to control the dispersion of the process
- ☑ These two charts must be used together



Setting Chart Limits

For $\bar{x}$ -Charts when we know the standard deviation

$$\text{Upper control limit (UCL)} = \bar{\bar{x}} + z\sigma_{\bar{x}}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{x}} - z\sigma_{\bar{x}}$$

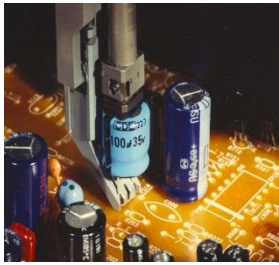
where $\bar{\bar{x}}$ = the average of the sample means

z = standard normal variable (2 for 95.44% confidence, 3 for 99.74% confidence)

$\sigma_{\bar{x}}$ = standard deviation of the distribution of sample means, computed as $\sigma/\sqrt{n}$

σ = population (process) standard deviation

n = sample size (number of observations per sample)



Setting Control Limits

Hour 1	
Sample Number	Weight of Oat Flakes
1	17
2	13
3	16
4	18
5	17
6	16
7	15
8	17
9	16
Mean 16.1	
$\sigma = 1$	

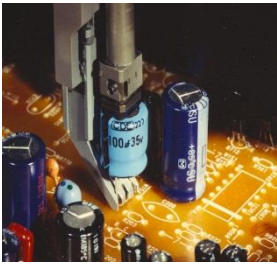
n = 9

Hour	Mean	Hour	Mean
1	16.1	7	15.2
2	16.8	8	16.4
3	15.5	9	16.3
4	16.5	10	14.8
5	16.5	11	14.2
6	16.4	12	17.3

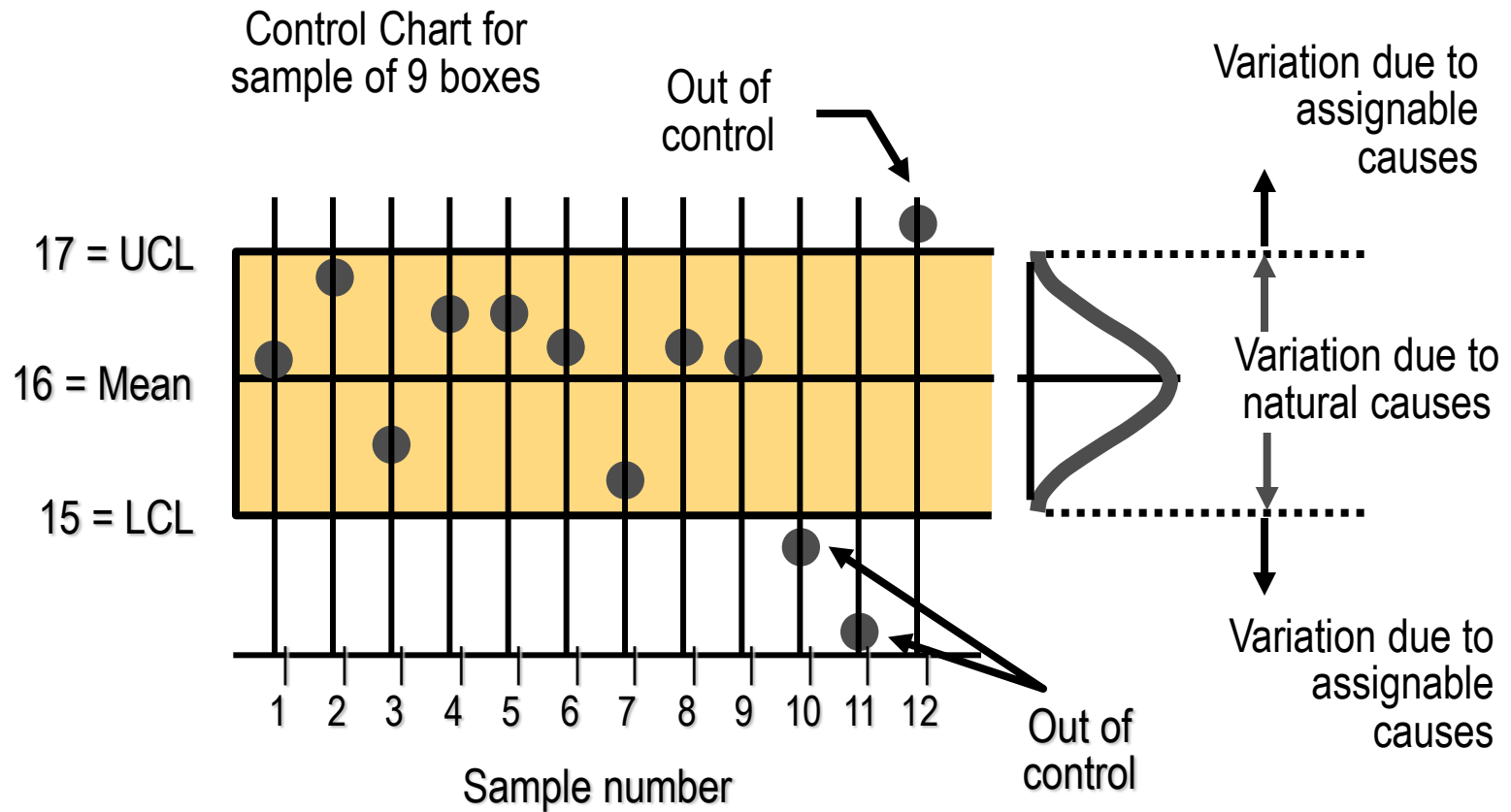
For 99.73% control limits, $z = 3$

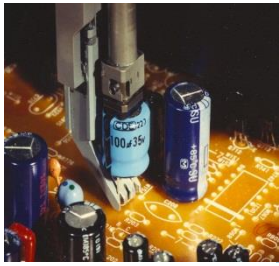
$$UCL_x = \bar{x} + z\sigma_x = 16 + 3(1/3) = 17 \text{ ozs}$$

$$LCL_x = \bar{x} - z\sigma_x = 16 - 3(1/3) = 15 \text{ ozs}$$



Setting control limits





Setting Chart Limits

For $\bar{X}$ -Charts when we don't know the standard deviation

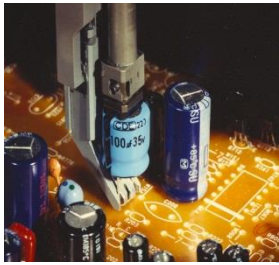
$$\text{Upper control limit (UCL)} = \bar{\bar{X}} + A_2 \bar{R}$$

$$\text{Lower control limit (LCL)} = \bar{\bar{X}} - A_2 \bar{R}$$

where $\bar{\bar{X}}$ = average of the sample means

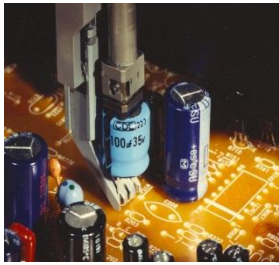
$\bar{R}$ = average range of the samples

A_2 = factor obtained from Table



Control Chart Factors

Sample Size n	Mean Factor A_2	Upper Range D_4	Lower Range D_3
2	1.880	3.268	0
3	1.023	2.574	0
4	.729	2.282	0
5	.577	2.115	0
6	.483	2.004	0
7	.419	1.924	0.076
8	.373	1.864	0.136
9	.337	1.816	0.184
10	.308	1.777	0.223
12	.266	1.716	0.284

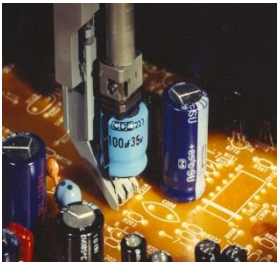


Setting Control Limits

Process average $\bar{x} = 16.01$ ounces

Average range $R = .25$

Sample size $n = 5$



Setting Control Limits

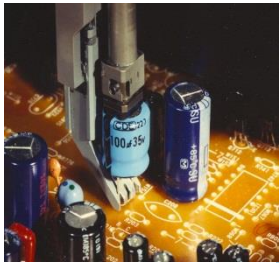
Process average $\bar{\bar{x}} = 16.01$ ounces

Average range $\bar{R} = .25$

Sample size $n = 5$

$$\begin{aligned} UCL_x &= \bar{\bar{x}} + A_2 \bar{R} \\ &= 16.01 + (.577)(.25) \\ &= 16.01 + .144 \\ &= 16.154 \text{ ounces} \end{aligned}$$

From Table



Setting Control Limit

Process average $\bar{x} = 16.01$ ounces

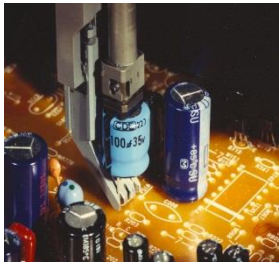
Average range $R = .25$

Sample size $n = 5$

$$\begin{aligned}UCL_x &= \bar{x} + A_2R \\&= 16.01 + (.577)(.25) \\&= 16.01 + .144 \\&= 16.154 \text{ ounces}\end{aligned}$$

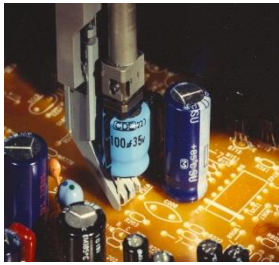
$$\begin{aligned}LCL_x &= \bar{x} - A_2R \\&= 16.01 - .144 \\&= 15.866 \text{ ounces}\end{aligned}$$

UCL = 16.154
Mean = 16.01
LCL = 15.866



R-Chart

- ☑ Type of variables control chart
- ☑ Shows sample ranges over time
 - ☑ Difference between smallest and largest values in sample
- ☑ Monitors process variability
- ☑ Independent from process mean



Setting Chart Limits

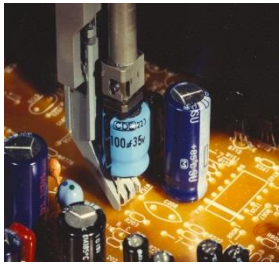
For R-Charts

$$CL = \bar{R}$$

$$UCL = D_4 \bar{R}$$

$$LCL = D_3 \bar{R}$$

where values for D_4 and D_3 are obtained from Table



Setting Control Limits

Average range $R = 5.3$ pounds

Sample size $n = 5$

From Table S6.1 $D_4 = 2.115$, $D_3 = 0$

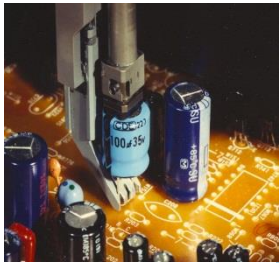
$$\begin{aligned} UCL_R &= D_4 R \\ &= (2.115)(5.3) \\ &= 11.2 \text{ pounds} \end{aligned}$$

$$\begin{aligned} LCL_R &= D_3 R \\ &= (0)(5.3) \\ &= 0 \text{ pounds} \end{aligned}$$

UCL = 11.2

Mean = 5.3

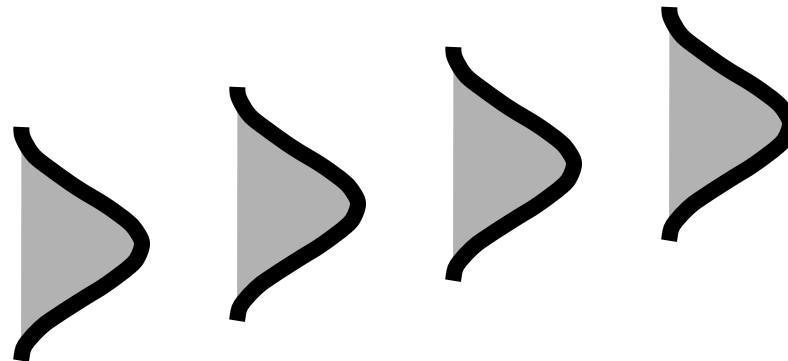
LCL = 0



Mean and Range Charts

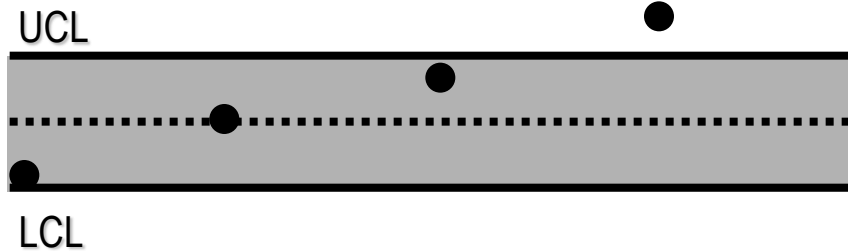
(a)

These sampling distributions result in the charts below



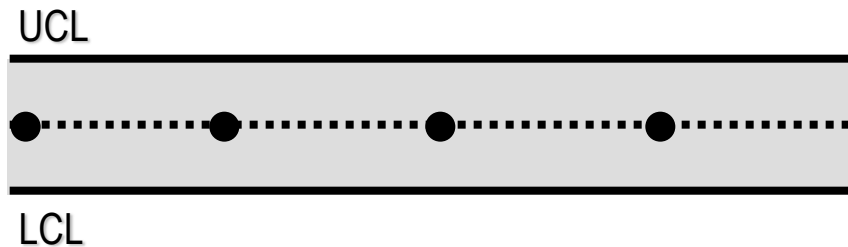
(Sampling mean is shifting upward but range is consistent)

$\bar{x}$ -chart



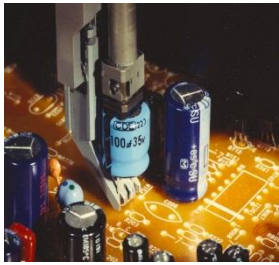
($\bar{x}$ -chart detects shift in central tendency)

R-chart



(R-chart does not detect change in mean)

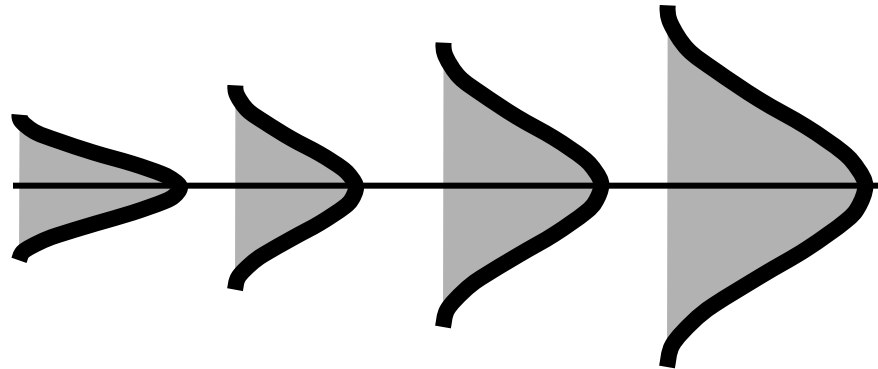
Figure S6.5



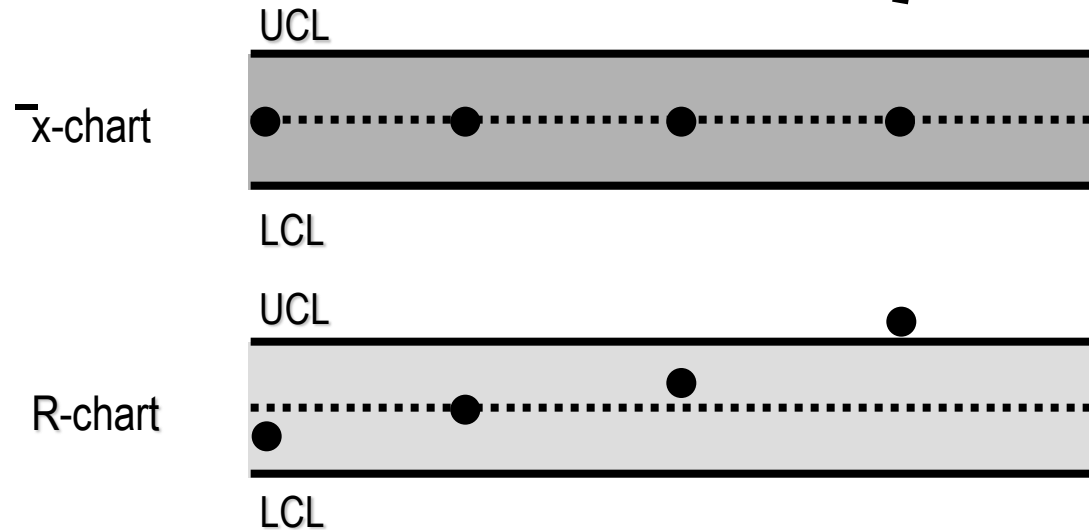
Mean and Range Charts

(b)

These sampling distributions result in the charts below



(Sampling mean is constant but dispersion is increasing)



($\bar{x}$ -chart does not detect the increase in dispersion)

(R-chart detects increase in dispersion)

Figure S6.5