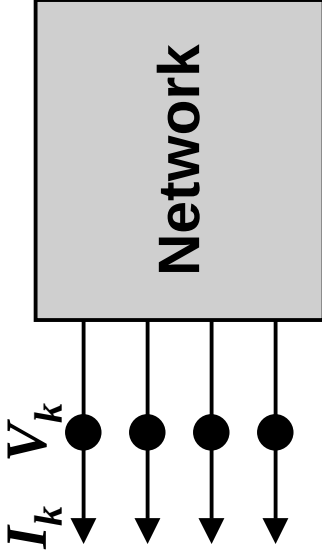


The Bus Admittance Matrix

- The matrix equation for relating the nodal voltages to the currents that flow into and out of a network using the admittance values of circuit branches

$$\mathbf{I}_{inj} = \mathbf{Y}_{bus} \cdot \mathbf{V}_{node}$$


- Used to form the network model of an interconnected power system

- ◆ Nodes represent substation bus bars
- ◆ Branches represent transmission lines and transformers
- ◆ Injected currents are the flows from generator and loads

Power Systems I

The Bus Admittance Matrix

- **Constructing the Bus Admittance Matrix (or the Y bus matrix)**

- ◆ form the nodal solution based upon Kirchhoff's current law

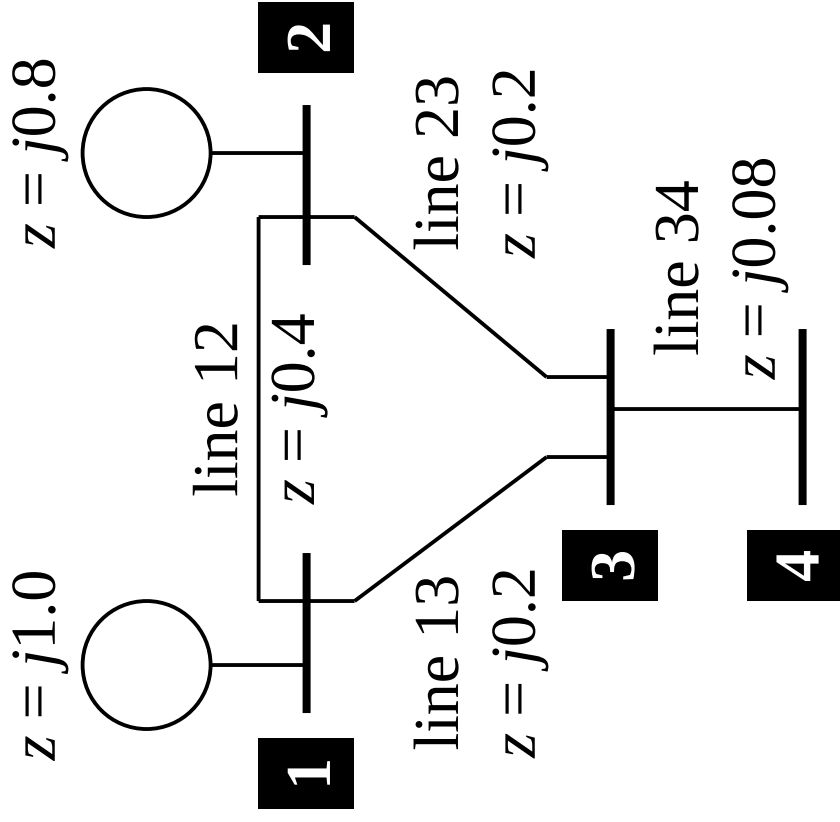
$$I_{k-inj} = y_{k0} V_k + y_{k1} (V_k - V_1) + y_{k2} (V_k - V_2) + \dots + y_{kn} (V_k - V_n)$$

- ◆ impedances are converted to admittances

$$y_{ij} = \frac{1}{z_{ij}} = \frac{1}{r_{ij} + j x_{ij}}$$

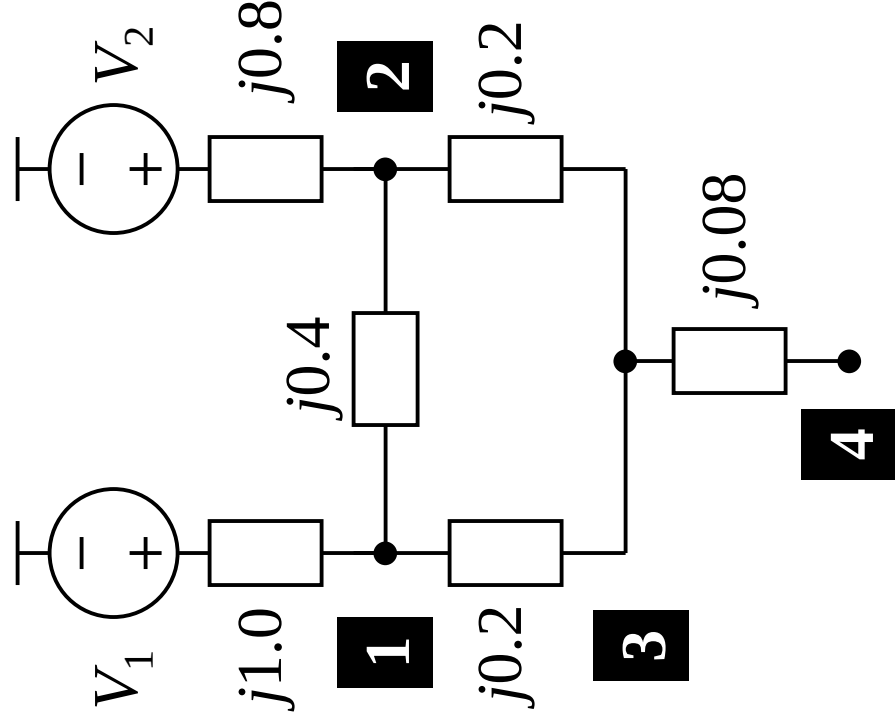
Matrix Formation Example

generator 1 generator 2



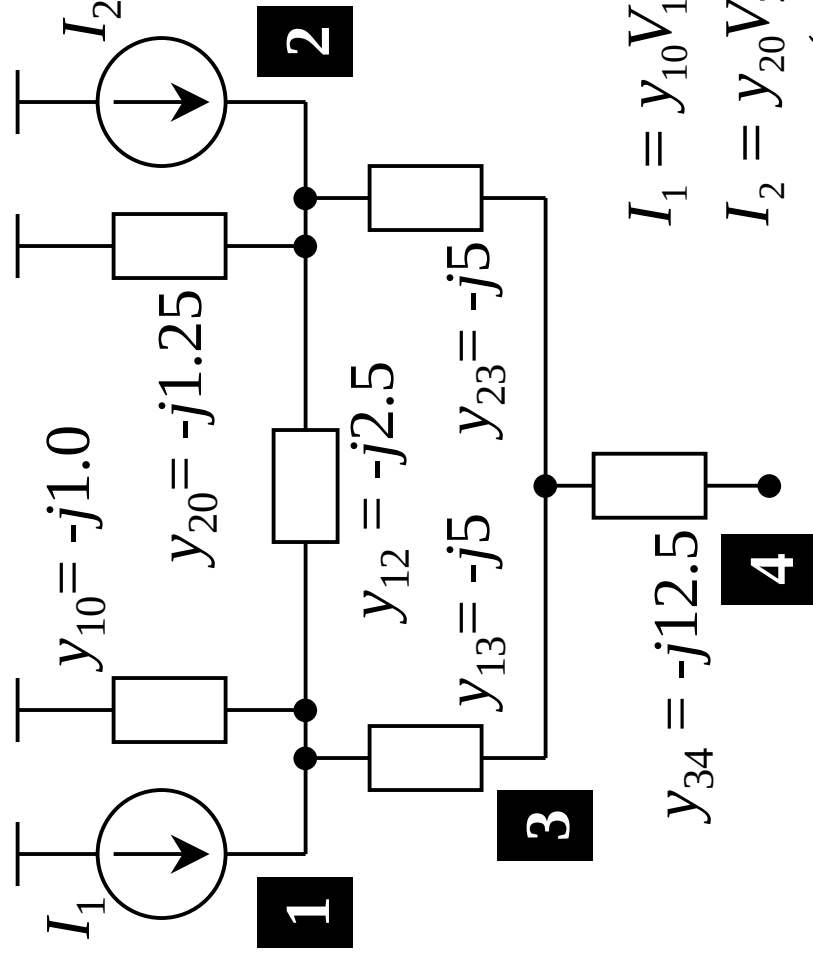
Network Diagram

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Impedance Diagram

Matrix Formation Example



KCL Equations

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3)$$

$$I_2 = y_{20}V_2 + y_{21}(V_2 - V_1) + y_{23}(V_2 - V_3)$$

$$0 = y_{31}(V_3 - V_1) + y_{32}(V_3 - V_2) + y_{34}(V_3 - V_4)$$

$$0 = y_{43}(V_4 - V_3)$$

Admittance Diagram

Power Systems I

Matrix Formation Example

Rearranging the KCL Equations

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3$$

$$I_2 = -y_{21}V_1 + (y_{20} + y_{21} + y_{23})V_2 - y_{23}V_3$$

$$0 = -y_{31}V_1 - y_{32}V_2 + (y_{31} + y_{32} + y_{34})V_3 - y_{34}V_4$$

$$0 = -y_{43}V_3 + y_{43}V_4$$

Matrix Formation of the Equations

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} (y_{10} + y_{12} + y_{13}) & -y_{12} & 0 \\ -y_{21} & (y_{20} + y_{21} + y_{23}) & 0 \\ -y_{31} & -y_{32} & (y_{31} + y_{32} + y_{34}) \\ 0 & 0 & -y_{43} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

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Matrix Formation Example

Completed Matrix Equation

$$Y_{11} = (y_{10} + y_{12} + y_{13}) = -j8.50 \quad Y_{23} = Y_{32} = -y_{23} = j5.00$$

$$Y_{12} = Y_{21} = -y_{12} = j2.50 \quad Y_{33} = (y_{31} + y_{32} + y_{34}) = -j22.50$$

$$Y_{13} = Y_{31} = -y_{13} = j5.00 \quad Y_{34} = Y_{43} = -y_{34} = j12.50$$

$$Y_{22} = (y_{20} + y_{21} + y_{23}) = -j8.75 \quad Y_{44} = y_{34} = -j12.50$$

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j8.50 & j2.50 & j5.00 & 0 \\ j2.50 & -j8.75 & j5.00 & 0 \\ j5.00 & j5.00 & -j22.50 & j12.50 \\ 0 & 0 & j12.50 & -j12.50 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Power Systems I

Y-Bus Matrix Building Rules

- Square matrix with dimensions equal to the number of buses
- Convert all network impedances into admittances
- Diagonal elements:
$$Y_{ii} = \sum_{j=0}^n y_{ij} \quad j \neq i$$
- Off-diagonal elements:
$$Y_{ij} = Y_{ji} = -y_{ij}$$
- Matrix is symmetrical along the leading diagonal

Example

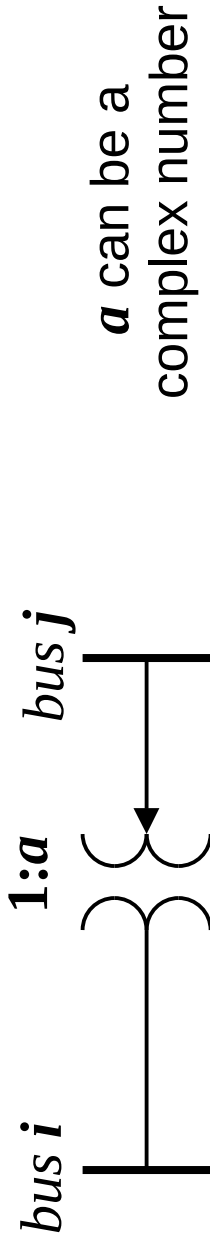
System Data

Line	Start	End	X value
g1	1	0	1.00
g2	5	0	1.25
L1	1	2	0.40
L2	1	3	0.50
L3	2	3	0.25
L4	2	5	0.20
L5	3	4	0.125
L6	4	5	0.50

Power Systems I

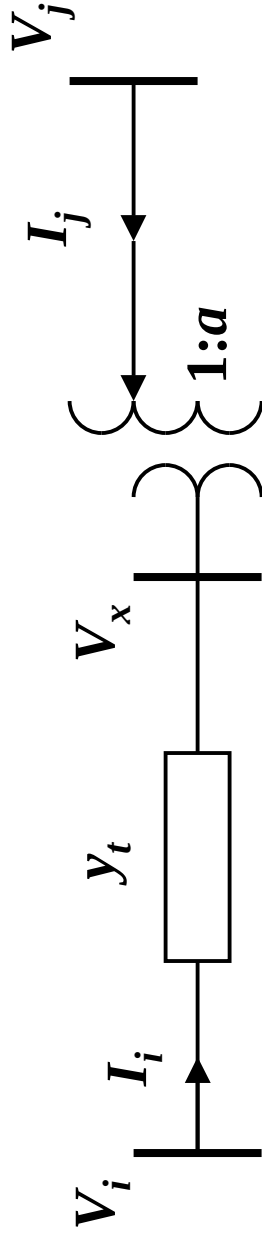
Tap-Changing Transformers

- The tap-changing transform gives some control of the power network by changing the voltages and current magnitudes and angles by small amounts
 - ♦ The flow of real power along a network branch is controlled by the angular difference of the terminal voltages
 - ♦ The flow of reactive power along a network branch is controlled by the magnitude difference of the terminal voltages
 - ♦ Real and reactive powers can be adjusted by voltage-regulating transformers and by phase-shifting transformers



Modeling of Tap-Changers

- ◆ the off-nominal tap ratio is given as $1:a$
- ◆ the nominal turns-ratio (N_1/N_2) was addressed with the conversion of the network to per unit
- ◆ the transformer is modeled as two elements joined together at a fictitious bus x



- ◆ basic circuit equations:

$$V_x = \frac{1}{a} V_j \quad I_i = -a^* \cdot I_j \quad I_i = y_t (V_i - V_x)$$

Modeling of Tap-Changers

- Making substitutions

$$V_x = \frac{1}{a} V_j \qquad I_i = y_t (V_i - V_x)$$

$$I_i = y_t \left(V_i - \frac{1}{a} V_j \right)$$

$$I_i = -a^* \cdot I_j$$

$$I_j = -\frac{1}{a^*} I_i$$

$$I_j = -\frac{y_t}{a^*} \left(V_i - \frac{1}{a} V_j \right) = -\frac{y_t}{a^*} V_i + \frac{y_t}{|a|^2} V_j$$

YBus Formation of Tap-Changers

- Matrix formation

$$I_i = \{y_t\}V_i + \left\{-\frac{y_t}{a}\right\}V_j$$

$$I_j = \left\{-\frac{y_t}{a}\right\}V_i + \left\{\frac{y_t}{|a|^2}\right\}V_j$$

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_t & -y_t/a \\ -y_t/a^* & y_t/|a|^2 \end{bmatrix} \cdot \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

Pi-Circuit Model of Tap-Changers

- Valid for real values of a
- Taking the y-bus formation, break the diagonal elements into two components
 - ◆ the off-diagonal element represent the impedance across the two buses
 - ◆ the remainder form the shunt element

