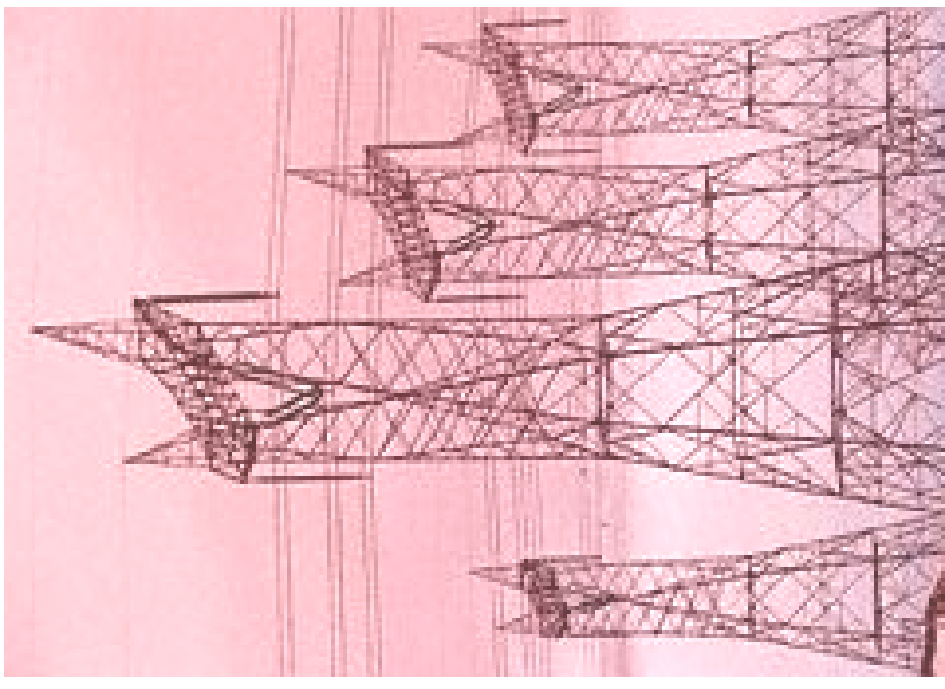


Transmission Lines

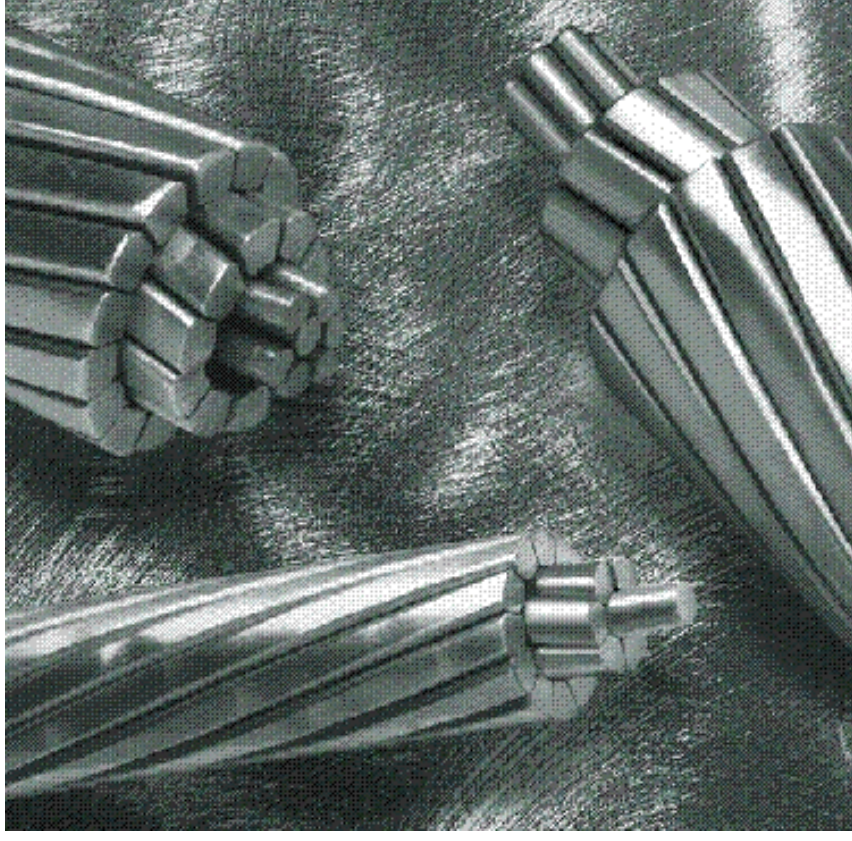
- Overhead Conductor
- Overhead Spacer Cable
- Underground Cable
- Three-Conductor Cable
- Service Cables



Overhead Conductors

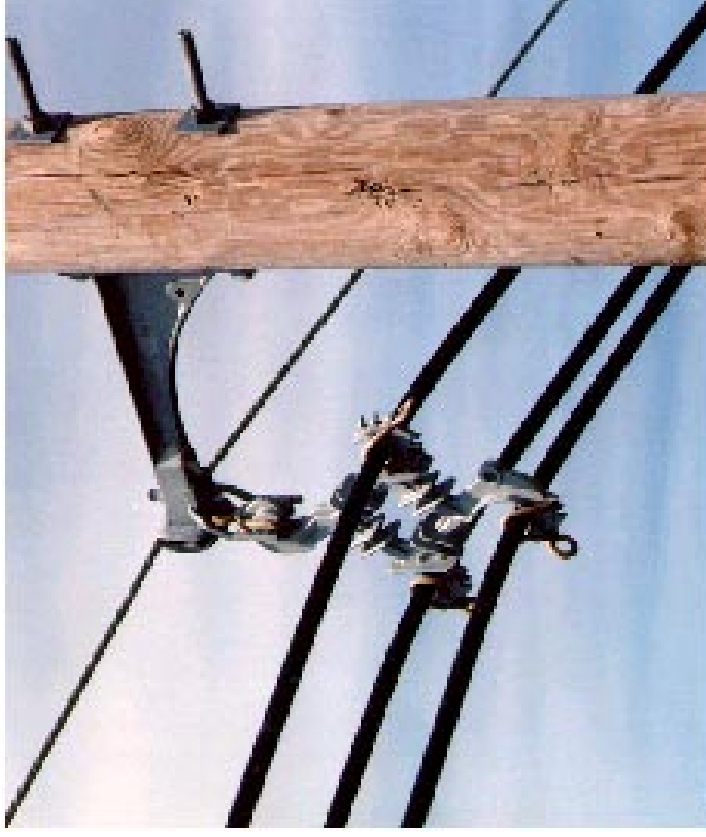
- **ACSR**
Aluminum Conductor with inner Steel Reinforced strands
- **ACAR**
Aluminum Conductor with inner Al alloy Reinforced strands
- **ACSR/AW**
Aluminum Conductor with inner Aluminoweld Steel Reinforced strands
- **Aluminum - current carrying member**
- **Steel - structural support**

Power Systems I

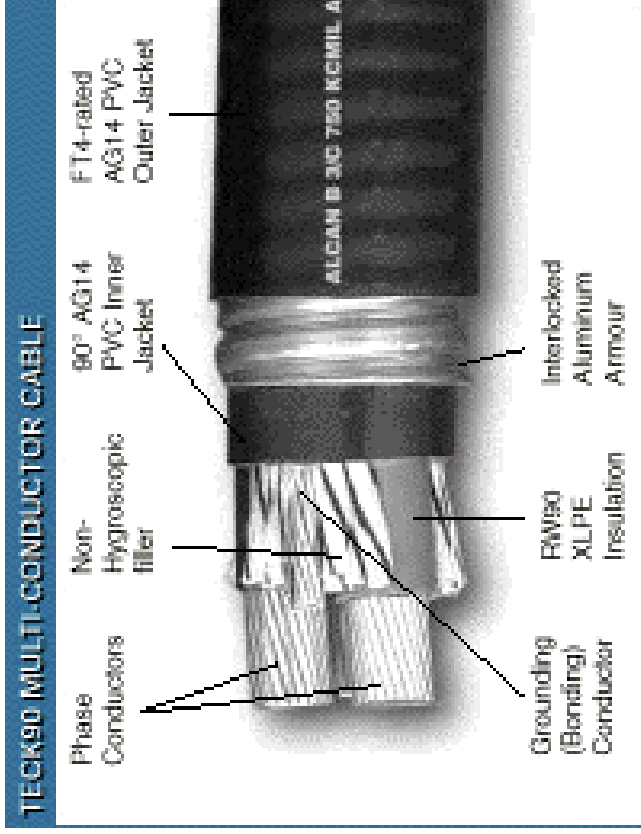
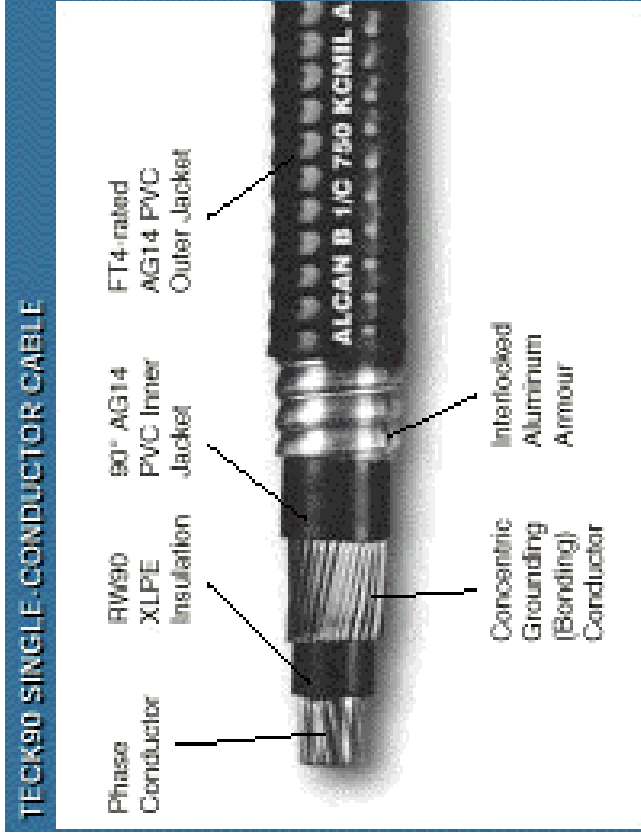


Overhead Cable

- Where conductor close proximity is required
- Insulating jacket surrounds each conductor
- Plastic spacers keep conductors from coming in contact with one another

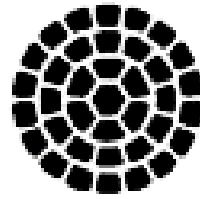


Cables



Specification

- CSA C22.2 No. 131 (TECK)
- CSA C22.2 No. 174 (Hazardous Locations)
- FT4-Rated: Vertical Flame Test - cable in cable tray (IEEE P1202)
- AG14 inner and outer PVC jacket:
 - Maximum 14% acid gas emission by weight
- FMRC Class 3912 Fire Test GP-2 (jacketed) GP-1 (unjacketed)



Power Systems I

Cables

- **Underground transmission and distribution cables**
- **Semiconducting material surrounds the conductor to grade the electric field**
- **Plastic jacket provides insulation and protection**
- **Neutral strands for an outer shell for protection and return currents**



Transmission Line Parameters

- **Line resistance**

- ◆ dc resistance

$$R_{dc} = \frac{\rho l}{A}$$

ρ = conductor resistivity

l = conductor length

- ◆ ac resistance

- skin effect

- at 60 Hz:

$$R_{ac} = 1.02 \cdot R_{dc}$$

A = conductor cross-sectional area

- **Temperature effects**

- ◆ increased resistance at conductor temperature rises
- ◆ wiring is rated for 65°C, 75°C, or 90°C
- ◆ ambient temperature is 20°C

$$R_{new} = R_{old} \frac{T + t_{new}}{T + t_{old}}$$

$$T_{Al} = 228^{\circ}\text{C}$$

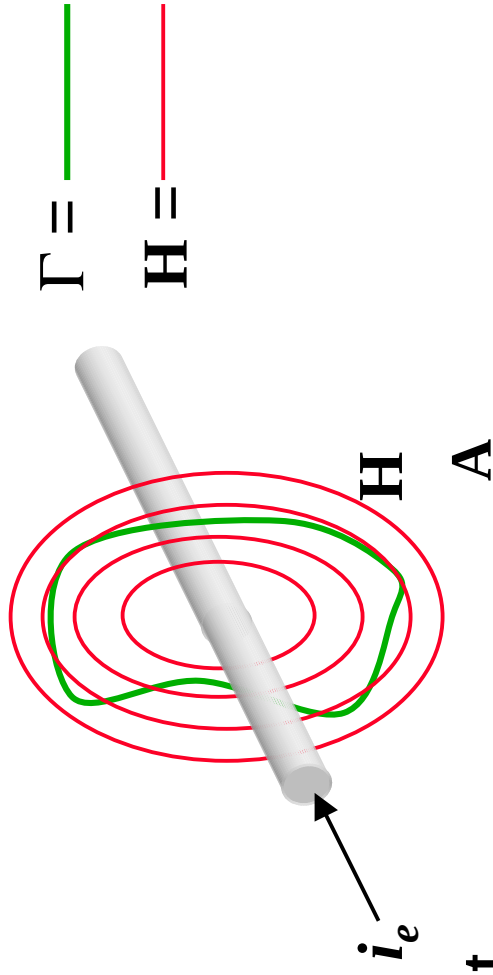
Power Systems I

Review of Magnetism and Inductance

Ampere's circuital law

$$F = \oint_{\Gamma} \mathbf{H} \cdot d\mathbf{l} = i_e$$

Integral of the scalar product of a closed path and the magnetic field equals the encircled current



$$\Gamma =$$

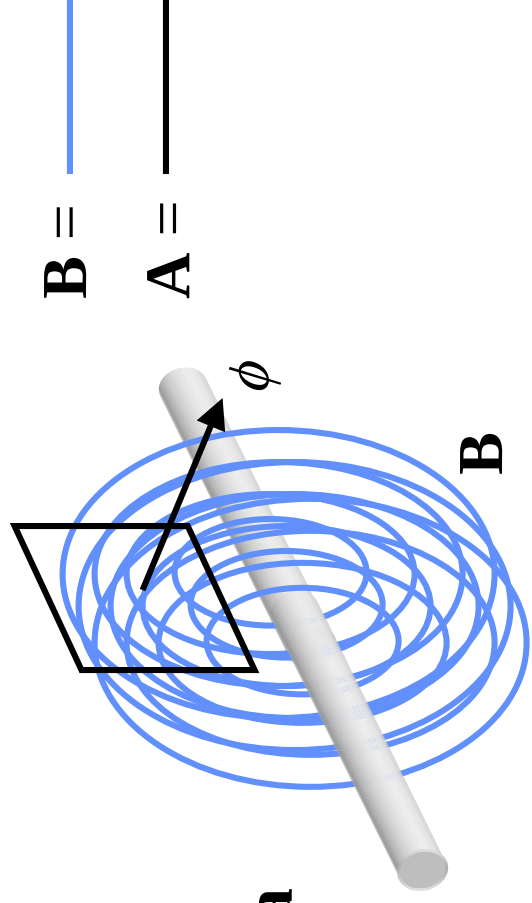
$$\mathbf{H} =$$

Magnetic Flux

$$\mathbf{B} = \mu \mathbf{H}$$

Integral of the flux density that is normal to a defined area

$$\phi = \int_A \mathbf{B} \cdot d\mathbf{a}$$



$$\mathbf{B} =$$

$$A =$$

Power Systems I

Review of Magnetism and Inductance

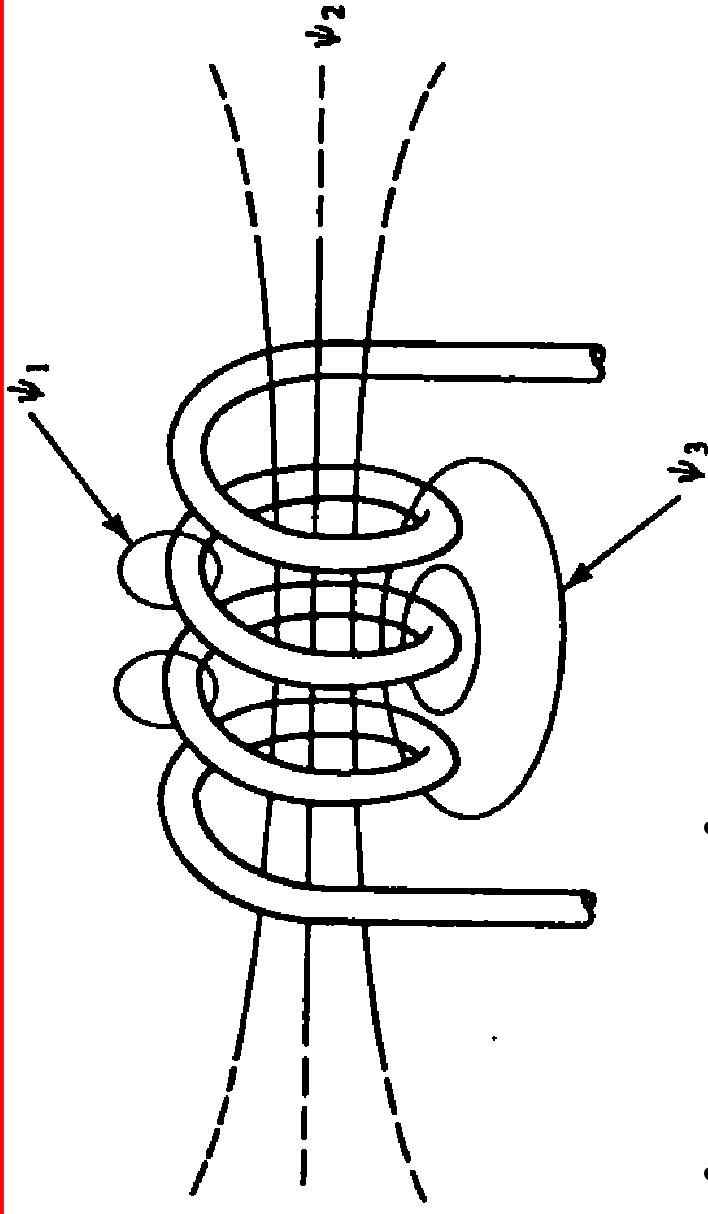
Flux Linkage

$$\lambda = \sum_{i=1}^N \phi_i$$

Inductance

$$L = \frac{\lambda}{I}$$

$$= \frac{\sum \phi}{I} = \frac{\sum \int \mathbf{B} \cdot d\mathbf{a}}{I} = \frac{\sum \int \mu \mathbf{H} \cdot d\mathbf{a}}{I}$$



Power Systems I

Inductance of a Single Conductor

- **Conditions:**

- ◆ infinite straight wire is an approximation of a reasonably long wire

- **Assumptions:**

- ◆ Image the wire to close at +/- infinity, establishing a kind of “one-turn coil” with the return path at infinity
- ◆ Straight infinitely long wire of radius r
- ◆ Uniform current density in the wire. Total current is I_x
- ◆ Flux lines form concentric circles (i.e. $\mathbf{H}$ is tangential)
- ◆ Angular symmetry - it suffices to consider H_x

Inductance of a Single Conductor

- **General:**
$$\int_0^{2\pi x} H_x \cdot dl = I_x \Rightarrow H = \frac{I_x}{2\pi x}$$
- **Case 1: Points inside of the conductor ($x < r$)**

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2} \rightarrow H_x = \frac{I}{2\pi r^2} x \rightarrow B_x = \frac{\mu_0 I}{2\pi r^2} x$$

$$d\phi_x = B_x dx = \frac{\mu_0 I}{2\pi r^2} x dx \rightarrow d\lambda_x = \frac{x^2}{r^2} d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx$$

$$\lambda_{int} = \int_0^r d\lambda_x = \frac{\mu_0 I}{2\pi r^4} \int_0^r x^3 dx = \frac{\mu_0 I}{8\pi} \rightarrow L_{int} = \frac{\mu_0}{8\pi} = 0.5 \times 10^{-7}$$

Inductance of a Single Conductor

- Case 2: Points outside of the conductor ($x > r$)

$$I_x = I \rightarrow B_x = \mu_0 H_x = \frac{\mu_0 I}{2\pi x}$$

$$d\phi_x = B_x dx = \frac{\mu_0 I}{2\pi x} dx \rightarrow d\lambda_x = d\phi_x = \frac{\mu_0 I}{2\pi x} dx$$

$$\lambda_{\text{ext}} = \int_{D_1}^{D_2} d\lambda_x = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{1}{x} dx = \frac{\mu_0 I}{2\pi} \ln \frac{D_2}{D_1} \rightarrow L_{\text{ext}} = 2 \times 10^{-7} \ln \frac{D_2}{D_1}$$

Inductance of a Single-Phase Line

- conductors of radii r_1 and r_2 , separated by a distance D

$$L_{1(\text{ext})} = 2 \times 10^{-7} \ln \frac{D}{r_1}$$

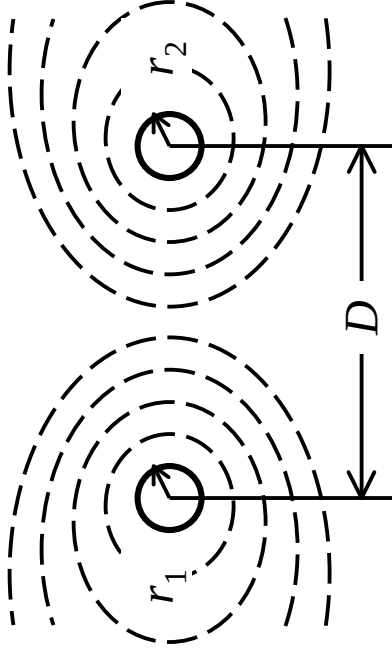
$$L_1 = L_{1(\text{int})} + L_{1(\text{ext})} = 0.5 \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D}{r_1}$$

$$L_1 = 2 \times 10^{-7} \left(\ln \frac{1}{r_1 e^{-1/4}} + \ln D \right)$$

$$r_1 = r_2 \quad L_1 = L_2 = L \quad r' = r e^{-1/4} = D_s$$

$$L = 2 \times 10^{-7} \left(\ln \frac{D}{r e^{-1/4}} \right) = 2 \times 10^{-7} \left(\ln \frac{D}{r'} \right) = 2 \times 10^{-7} \left(\ln \frac{D}{D_s} \right)$$

Power Systems I



Flux Linkage - Self and Mutual Inductances

From the 2 conductor case:

$$\begin{array}{lcl} \lambda_1 = L_{11}I_1 + L_{12}I_2 & I_1 = -I_2 \rightarrow & \lambda_1 = L_{11}I_1 - L_{12}I_1 \\ \lambda_2 = L_{21}I_1 + L_{22}I_2 & & \lambda_2 = -L_{21}I_2 + L_{22}I_2 \end{array}$$

$$\begin{array}{lcl} L_{11} = 2 \times 10^{-7} \left(\ln \frac{1}{r_1'} \right) & L_{22} = 2 \times 10^{-7} \left(\ln \frac{1}{r_2'} \right) \\ L_{12} = L_{21} = -2 \times 10^{-7} (\ln D) = 2 \times 10^{-7} \left(\ln \frac{1}{D} \right) \end{array}$$

Power Systems I

Total Inductance

General Case:

$$I_1 + I_2 + \dots + I_i + \dots + I_n = 0$$

$$\lambda_i = L_{ii}I_i + \sum_{j=1}^n L_{ij}I_j \quad j \neq i$$

$$\lambda_i = 2 \times 10^{-7} \left(I_i \ln \frac{1}{r_i'} + \sum_{j=1}^n I_j \frac{1}{D_{ij}} \right) \quad j \neq i$$

Power Systems I

Inductance of Three-Phase Lines

- Symmetrical spacing

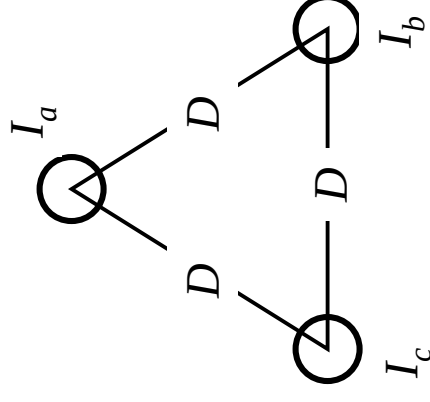
$$I_a + I_b + I_c = 0$$

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D} \right)$$

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right)$$

$$\lambda_a = 2 \times 10^{-7} I_a \ln \frac{D}{r'}$$

$$L = 0.2 \ln \frac{D}{D_s}$$



Inductance of Three-Phase Lines

- Asymmetrical spacing

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{12}} + I_c \ln \frac{1}{D_{13}} \right)$$

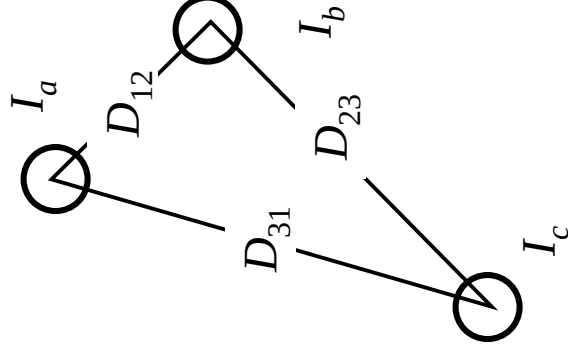
$$\lambda_b = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{21}} + I_c \ln \frac{1}{D_{23}} \right)$$

$$\lambda_c = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{31}} + I_c \ln \frac{1}{D_{32}} \right)$$

$$\lambda = \mathbf{L}\mathbf{I}$$

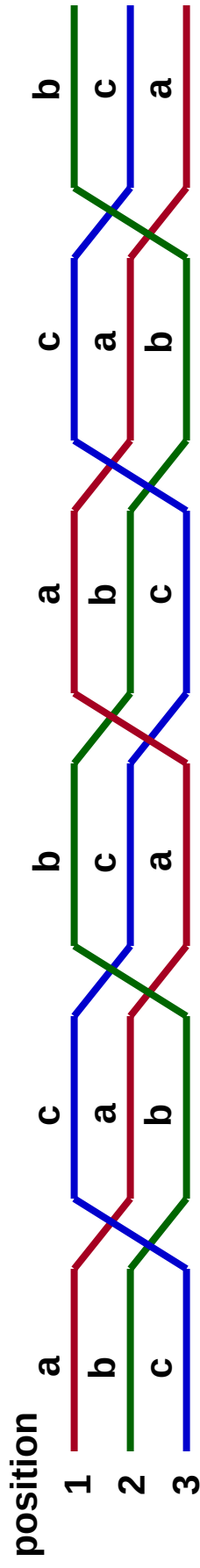
$$L = 2 \times 10^{-7} \ln \begin{bmatrix} \frac{1}{r'} & \frac{1}{D_{12}} & \frac{1}{D_{13}} \\ \frac{1}{D_{21}} & \frac{1}{r'} & \frac{1}{D_{23}} \\ \frac{1}{D_{31}} & \frac{1}{D_{32}} & \frac{1}{r'} \end{bmatrix}$$

Power Systems I



Transposition

- **The practice of equilateral arrangement of phases is not convenient**
 - ◆ horizontal or vertical configurations are most popular
 - ◆ Symmetry is lost - unbalanced conditions
- **restore balanced conditions by the method of transposition of lines**
 - ◆ Average inductance of each phase will be the same



- ◆ Each phase occupies each position for the same fraction of the total length of the line

Power Systems I

Review of Electric Fields

Gauss's law

$$q_e = \int_A \mathbf{D} \cdot d\mathbf{a}$$

Electric field

$$\mathbf{D} = \varepsilon \mathbf{E}$$

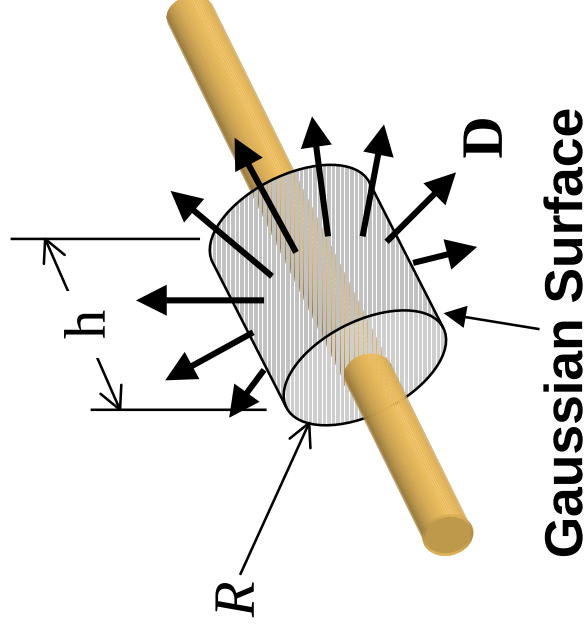
Electric field

$$V_{12} = V_{D_1} - V_{D_2} = - \int_{D_1}^{D_2} \mathbf{E} \cdot d\mathbf{l}$$

Capacitance

$$q = C v$$

Power Systems I



Infinite Straight Wire

$$V_{12} = \int_{D_1}^{D_2} \frac{q}{2\pi\epsilon_0 x} dx = \frac{q}{2\pi\epsilon_0} \ln \frac{D_2}{D_1}$$

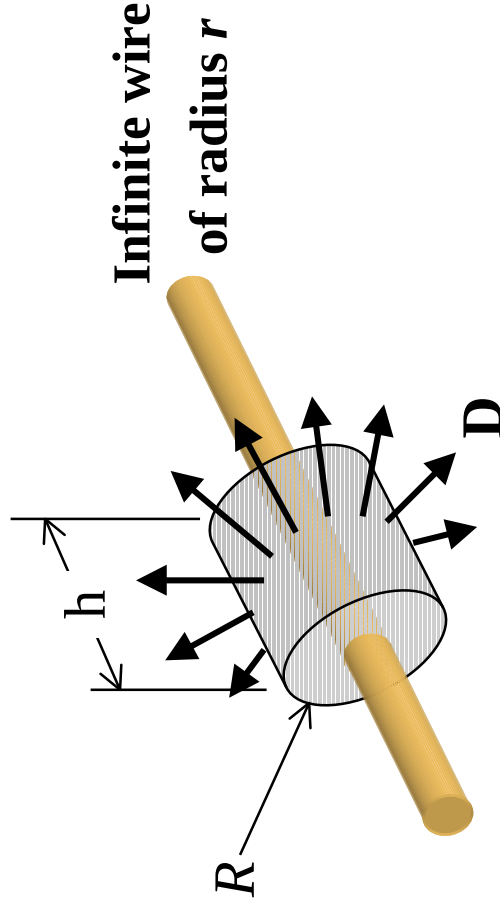
$$V_{12(q1)} = \frac{q_1}{2\pi\epsilon_0} \ln \frac{D}{r}$$

$$V_{21(q2)} = \frac{q_2}{2\pi\epsilon_0} \ln \frac{r}{D}$$

$$V_{12} = V_{12(q1)} + V_{21(q2)} = \frac{q_1}{2\pi\epsilon_0} \ln \frac{D}{r} + \frac{q_2}{2\pi\epsilon_0} \ln \frac{r}{D}$$

$$V_{12} = \frac{q}{\pi\epsilon_0} \ln \frac{D}{r}$$

Power Systems I

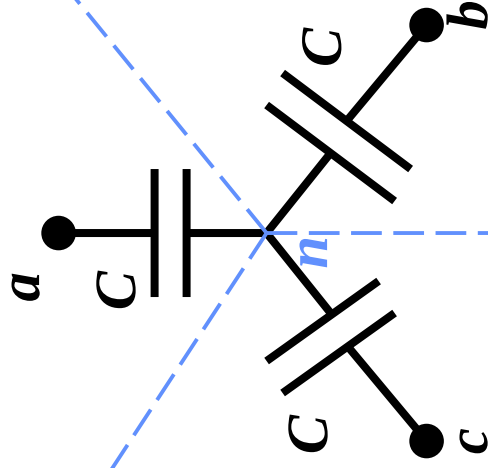


$$C = \frac{q}{v}$$

$$C = \frac{2\pi\epsilon}{\ln \frac{D}{r}}$$

Three-Phase Capacitance

- Equilateral spacing

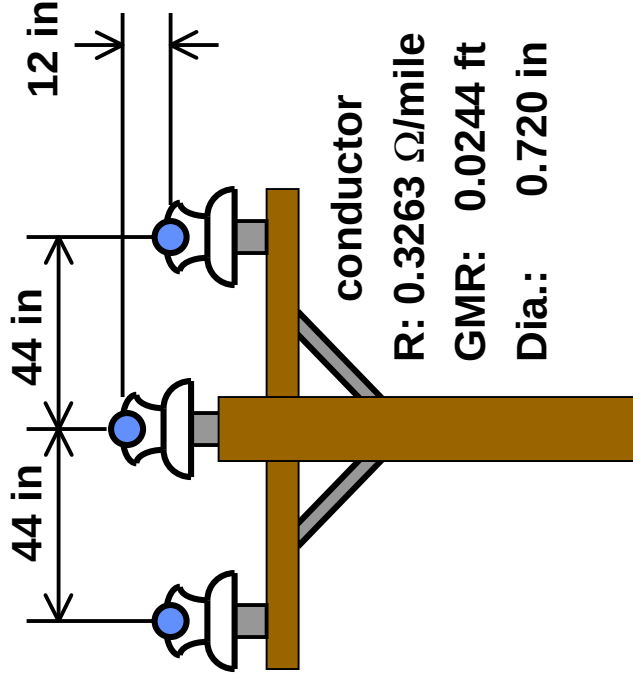


GMD_ϕ = geometric mean distance
between conductors
 r_ϕ = conductor radius

$$C = \frac{0.0389}{\log_{10} \left(\frac{GMD_\phi}{r_\phi} \right)} \text{ } \mu\text{F per mile per phase}$$

Example

Calculate the resistance, inductive reactance, and capacitive reactance per phase and rated current carrying capacity for the overhead line shown. Assume the line operates at 60 Hz



Power Systems I

$$GMD_{\phi} = \sqrt[3]{d_{12} d_{23} d_{13}} = \sqrt[3]{(45.6)(88)(45.6)}$$

$$= 56.8 \text{ in} = 4.73 \text{ ft}$$

$$Z_a = (0.3263) + j 0.2794 \frac{(60)}{60} \log_{10} \left(\frac{4.73}{0.0244} \right)$$

$$= 0.326 + j 0.639 \quad \Omega / \text{mi}$$

$$r_{\phi} = \frac{1}{2} \text{ dia} = \frac{1}{2} (0.720 \text{ in}) \cdot \frac{1}{12} = 0.03 \text{ ft}$$

$$C = \frac{0.0389}{\log_{10}(4.73/0.03)} = 0.177 \text{ } \mu\text{F}/\text{mi}/\text{phs}$$

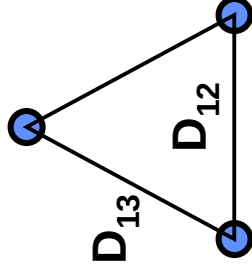
$$X_c = 1/(2\pi 60 \cdot 0.177 \text{ } \mu\text{F}) = 149.9 \quad \Omega \text{ mi}$$

Conductor Bundling

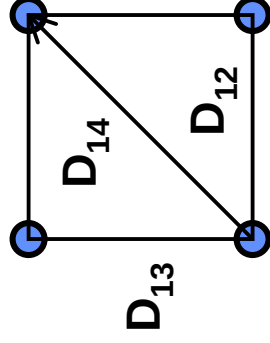
- Commonly used to reduce the electric field strength at the conductor surface
- Used on overhead lines above 230 kV
- Conductors are connected in parallel
- Typical bundled conductor configurations



2 conductors



3 conductors



4 conductors

Conductor Bundling

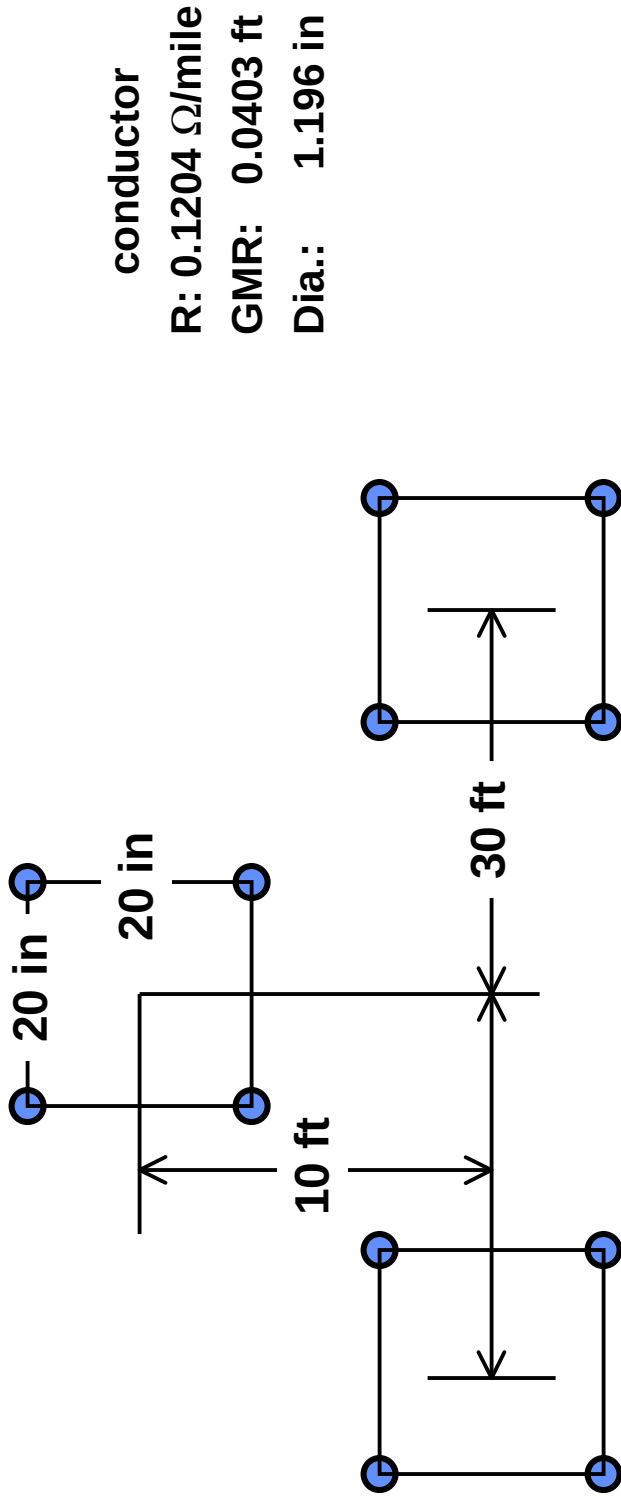
- The use of bundled conductors effects the impedance of the line, the GMR_{ϕ} , the GMD_{ϕ} , and the equivalent radius
- GMD_{ϕ} : the distance between the center of each bundle is used
- GMR_{ϕ} :

$$GMR'_{\phi} = \sqrt[n]{GMR_{\phi} \cdot \prod_{i=2}^n D_{1i}}$$

- Equivalent radius
$$r'_{\phi} = \sqrt[n]{r_{\phi} \cdot \prod_{i=2}^n D_{1i}}$$

Example

Calculate the resistance, inductive reactance, and capacitive reactance of the overhead line shown. Assume the line operates at 60 Hz



Example

$$R' = \frac{1}{4} \cdot 0.1204 = 0.0301 \quad \Omega / \text{mi}$$

$$GMD_{\phi} = \sqrt[3]{D_{12} D_{23} D_{13}} = \sqrt[3]{(31.6)(60)(31.6)} = 39.15 \text{ ft}$$

$$GMR_{\phi} = \sqrt[4]{(0.0403)(1.67)(1.414)(1.67)} = 0.7178 \text{ ft}$$

$$\begin{aligned} Z_a &= (0.0301) + j 0.2794 \frac{(60)}{60} \log_{10} \left(\frac{39.15}{0.7178} \right) \\ &= 0.0301 + j 0.485 \quad \Omega / \text{mi} \end{aligned}$$

$$r_{\phi} = \frac{1}{2} \text{ dia} = \frac{1}{2} (1.196 \text{ in}) \cdot \frac{1}{12} = 0.0498 \text{ ft}$$

$$r'_{\phi} = \sqrt[4]{(0.0498)(1.67)(1.414)(1.67)} = 0.7568 \text{ ft}$$

$$C = \frac{0.0389}{\log_{10}(39.15/0.7568)} = 0.0227 \quad \mu\text{F}/\text{mi}/\text{phs}$$

$$X_C = 1/(2\pi 60 \cdot 0.177 \mu\text{F}) = 116.85 \quad \Omega/\text{mi}$$

Power Systems I

Transmission Line Modeling

- **Transmission lines are represented by an equivalent circuit with parameters on a per-phase basis**
 - ◆ Voltages are expressed as phase-to-neutral
 - ◆ Currents are expressed for one phase
 - ◆ The three phase system is reduced to an equivalent single-phase
- **All lines are made up of distributed series inductance and resistance, and shunt capacitance and conductance**
 - ◆ Line parameters: R , L , C , & G
- **Three types of models**
 - ◆ depend on the length and the voltage level
 - ◆ short, medium, and long length line models

ABCD Two-Port Network

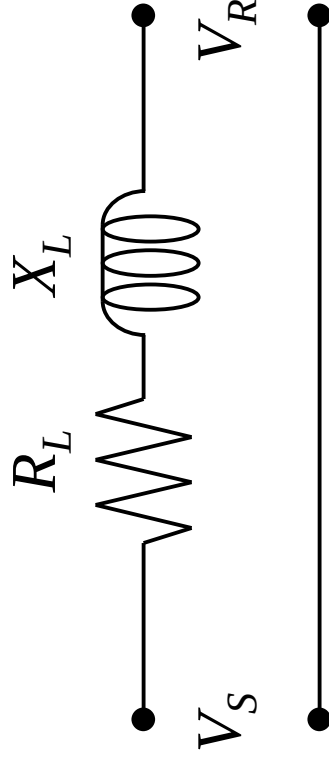
- All transmission line models may be described as a two-port network
- The ABCD two-port network is the most common representation
- The network is described by the four constants: A , B , C , & D
- **Network equations:**

- ◆ circuit equations
$$V_S = A V_R + B I_R$$
$$I_S = C V_R + D I_R$$
- ◆ matrix form
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Short Transmission Line Model

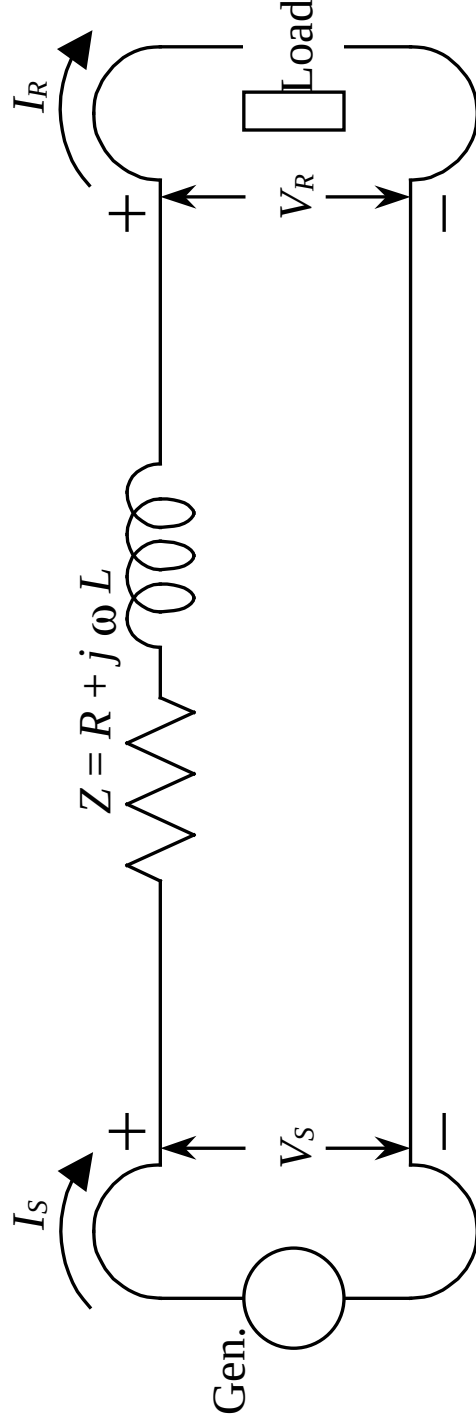
- The short transmission line model may be used when
 - ◆ The line length is less than 50 miles (80 km), or
 - ◆ The line voltage is not over 69 kV
- Modeling of the transmission line parameters
 - ◆ The shunt capacitance and conductance are ignored
 - ◆ The line resistance and reactance are treated as lumped parameters

- Circuit of the short model



Short Transmission Line Model

- Circuit analysis of the short line model



$$\begin{aligned} I_S &= I_R \\ V_S &= V_R + I_R (R + j\omega_s L) \\ &= V_R + I_R Z \end{aligned}$$

Two-Port Representation

- **Circuit Equations:**
$$V_S = V_R + Z_{line} I_R$$
$$I_S = I_R$$
 - ♦ Matrix representation:
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z_{line} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$
 - ♦ ABCD values:
$$A = 1$$
$$B = Z_{line}$$
$$C = 0$$
$$D = A = 1$$

Short Transmission Line Example

- 40 km, 220 kV transmission line has per phase
 - ◆ $R = 0.15 \Omega/\text{km}$ $L = 1.3263 \text{ mH}/\text{km}$
- Find V , S , $V.R.$, and η at the sending end of the line for
 - ◆ 381 MVA load at 0.8 lagging pf at 220 kV

$$Z = (r + j\omega L)\ell = (0.15\Omega + j2\pi \times 60 \times 1.3263 \times 10^{-3}) \cdot 40$$

$$Z = 6 + j20\Omega$$

$$V_R = \frac{220,000 \angle 0^\circ}{\sqrt{3}} = 127,000 \angle 0^\circ$$

$$S_{R(3\phi)} = 381 \angle (\cos^{-1} 0.8) = 381 \angle 36.9^\circ = 304.8 + j228.6 \text{ MVA}$$

Short Transmission Line Example

$$I_R = \frac{S_{R(3\phi)}^*}{3 V_R^*} = \frac{381 \times 10^6 \angle -36.9^\circ}{3 \times 127,000 \angle 0^\circ} = 1000 \angle -36.9^\circ \text{ A}$$

$$\begin{aligned} V_S &= V_R + Z I_R = 127,000 \angle 0^\circ + (6 + j20)(1000 \angle -36.9^\circ) \\ &= 144,330 \angle 4.93^\circ \end{aligned}$$

$$|V_{S-LL}| = \sqrt{3} \cdot |V_S| = 250 \text{ kV}$$

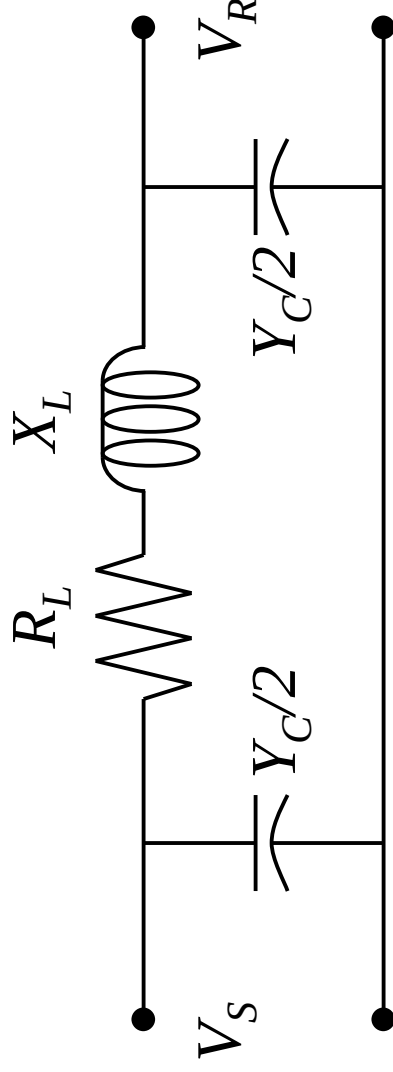
$$\begin{aligned} S_{S(3\phi)} &= 3 \cdot V_S \cdot I_S^* = 3 \cdot (144,330 \angle 4.93^\circ)(1000 \angle -36.9^\circ) \\ &= 322.8 + j288.6 = 433 \angle 41.8^\circ \text{ MVA} \end{aligned}$$

$$VR\% = \frac{250 - 220}{220} \times 100\% = 13.6\% \quad \eta = \frac{304.8}{322.8} \times 100\% = 94.4\%$$

Power Systems I

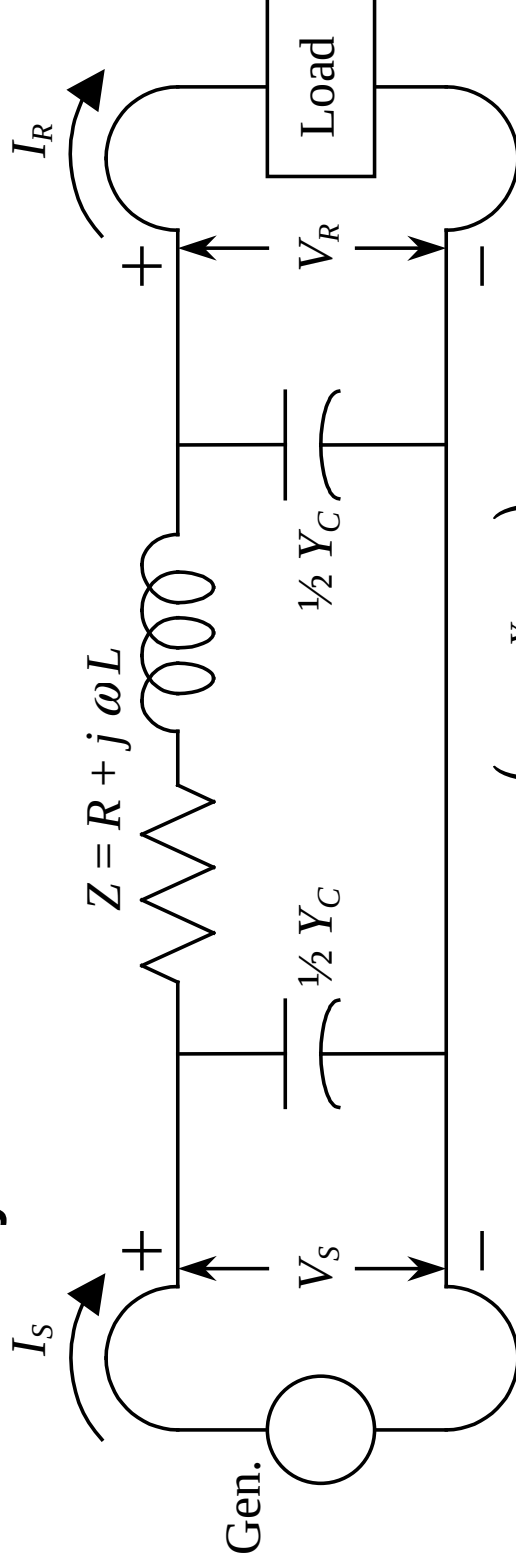
Medium Transmission Line Model

- **The medium transmission line model may be used when**
 - ◆ The line length is greater than 50 miles (80 km)
 - ◆ The line length is less than 150 miles (250 km)
- **Modeling of the transmission line parameters**
 - ◆ Half of the shunt capacitance is considered to be lumped at each end of the line
 - ◆ The line resistance and reactance are treated as lumped parameters
- **Circuit model:**



Medium Transmission Line Model

- Circuit analysis of the short line model



$$V_S = V_R + Z_{line} \left(I_R + \frac{Y_C}{2} V_R \right)$$

$$= \left(1 + \frac{Z_{line} Y_C}{2} \right) V_R + Z_{line} I_R$$

$$I_S = \left(I_R + \frac{Y_C}{2} V_R \right) + \frac{Y_C}{2} V_S$$

$$= Y_C \left(1 + \frac{Z_{line} Y_C}{4} \right) V_R + \left(1 + \frac{Z_{line} Y_C}{2} \right) I_R$$

Power Systems I

Two-Port Representation

- **Circuit Equations:**
$$V_S = V_R + Z_{line} \left(I_R + \frac{Y_C}{2} V_R \right)$$
$$= \left(1 + \frac{Z_{line} Y_C}{2} \right) V_R + Z_{line} I_R$$
$$I_S = \left(I_R + \frac{Y_C}{2} V_R \right) + \frac{Y_C}{2} V_S$$
$$= Y_C \left(1 + \frac{Z_{line} Y_C}{4} \right) V_R + \left(1 + \frac{Z_{line} Y_C}{2} \right) I_R$$
- ♦ **Matrix representation:**
$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 + \frac{Z_{line} Y_C}{2} & Z_{line} \\ Y_C \left(1 + \frac{Z_{line} Y_C}{4} \right) & 1 + \frac{Z_{line} Y_C}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$
- ♦ **ABCD values:**
$$A = 1 + \frac{Z_{line} Y_C}{2} \quad B = Z_{line}$$
$$C = Y_C \left(1 + \frac{Z_{line} Y_C}{4} \right) \quad D = 1 + \frac{Z_{line} Y_C}{2}$$

Medium Transmission Line Example

- 130 km, 345 kV transmission line has per phase
 - ♦ $R = 0.036 \Omega/\text{km}$ $L = 0.80 \text{ mH}/\text{km}$ $C = 0.0112 \mu\text{F}/\text{km}$
- Find **V** and **S** at the sending end of the line for
 - ♦ 270 MVA load at 0.8 lagging pf at 325 kV

$$\begin{aligned} Z &= (r + j\omega L)\ell = (0.036\Omega + j2\pi \times 60 \times 0.8 \times 10^{-3}) \cdot 130 \\ &= 4.68 + j39.2\Omega \end{aligned}$$

$$Y = (j\omega C)\ell = (j2\pi \times 60 \times 0.0112 \times 10^{-6}) \cdot 130 = j0.549 \text{ siemens}$$

$$V_R = \frac{325,000 \angle 0^\circ}{\sqrt{3}} = 187,600 \angle 0^\circ$$

$$S_{R(3\phi)} = 270 \angle (\cos^{-1} 0.8) = 270 \angle 36.9^\circ = 216 + j162 \text{ MVA}$$

Power Systems I

Medium Transmission Line Example

$$I_R = \frac{S_{R(3\phi)}^*}{3V_R^*} = \frac{270 \times 10^6 \angle -36.9^\circ}{3 \times 187,600 \angle 0^\circ} = 480 \angle -36.9^\circ \text{ A}$$

$$ABCD = \begin{bmatrix} 0.989 + j0.001284 & 4.68 + j39.2 \\ -3.53 \times 10^{-7} + j5.46 \times 10^{-4} & 0.989 + j0.001284 \end{bmatrix}$$

$$V_S = AV_R + BI_R = (187,600 \angle 0^\circ)(0.989 + j0.001284) + (480 \angle -36.9^\circ)(4.68 + j39.2)$$

$$= 199,160 \angle 4.02^\circ$$

$$I_S = CV_R + DI_R = (187,600 \angle 0^\circ)(-3.53 \times 10^{-7} + j5.46 \times 10^{-4}) + (480 \angle -36.9^\circ)(0.989 + j0.001284)$$

$$= 421.5 \angle -25.58^\circ$$

Power Systems I

Medium Transmission Line Example

$$|V_{S-LL}| = \sqrt{3} \cdot |V_S| = 345 \text{ kV}$$

$$\begin{aligned} S_{S(3\phi)} &= 3 \cdot V_S \cdot I_S^* = 3 \cdot (199,160 \angle 4.02^\circ) (421 \angle -25.58^\circ) \\ &= 218.9 + j124.2 \text{ MVA} \quad \text{pf} = 0.87 \end{aligned}$$

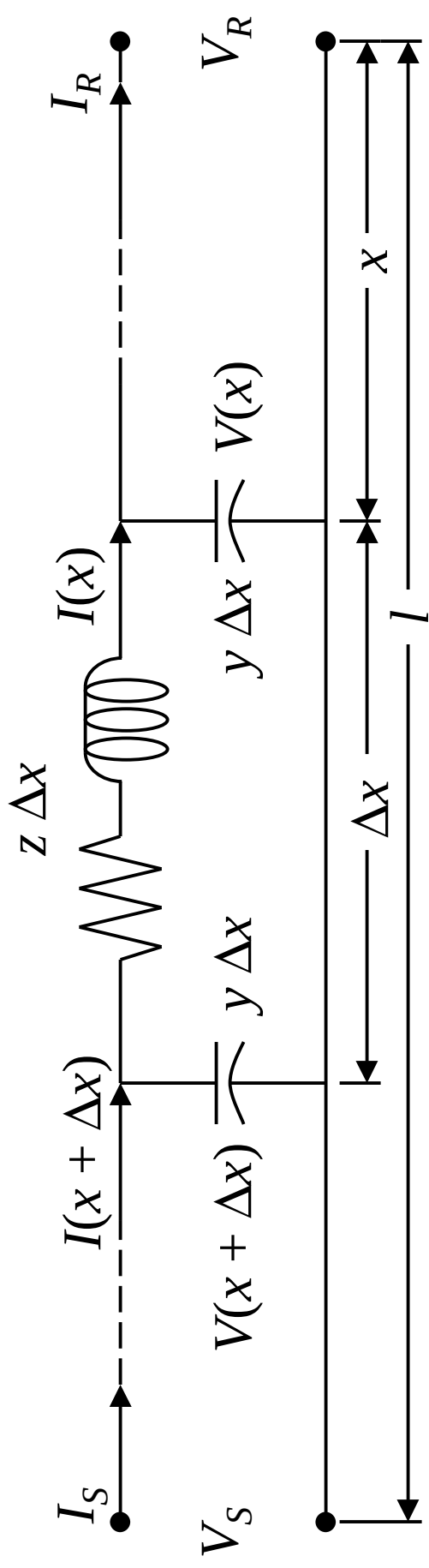
$$VR\% = \frac{V_{R(NL)} - V_{R(FL)}}{V_{R(FL)}} \times 100\% = \frac{V_{S(FL)} / A - V_{R(FL)}}{V_{R(FL)}} \times 100\%$$

$$\begin{aligned} &= \frac{345 / |0.989 + j0.001284| - 325}{325} \times 100\% = 7.3\% \end{aligned}$$

Long Transmission Line Model

- **The long transmission line model are used when**
 - ◆ The line length is greater than 150 miles (250 km)
- **Modeling of the transmission line parameters**
 - ◆ Accuracy obtained by using distributed parameters
 - ◆ The series impedance per unit length is z
 - ◆ The shunt admittance per unit length is y

Long Transmission Line Model

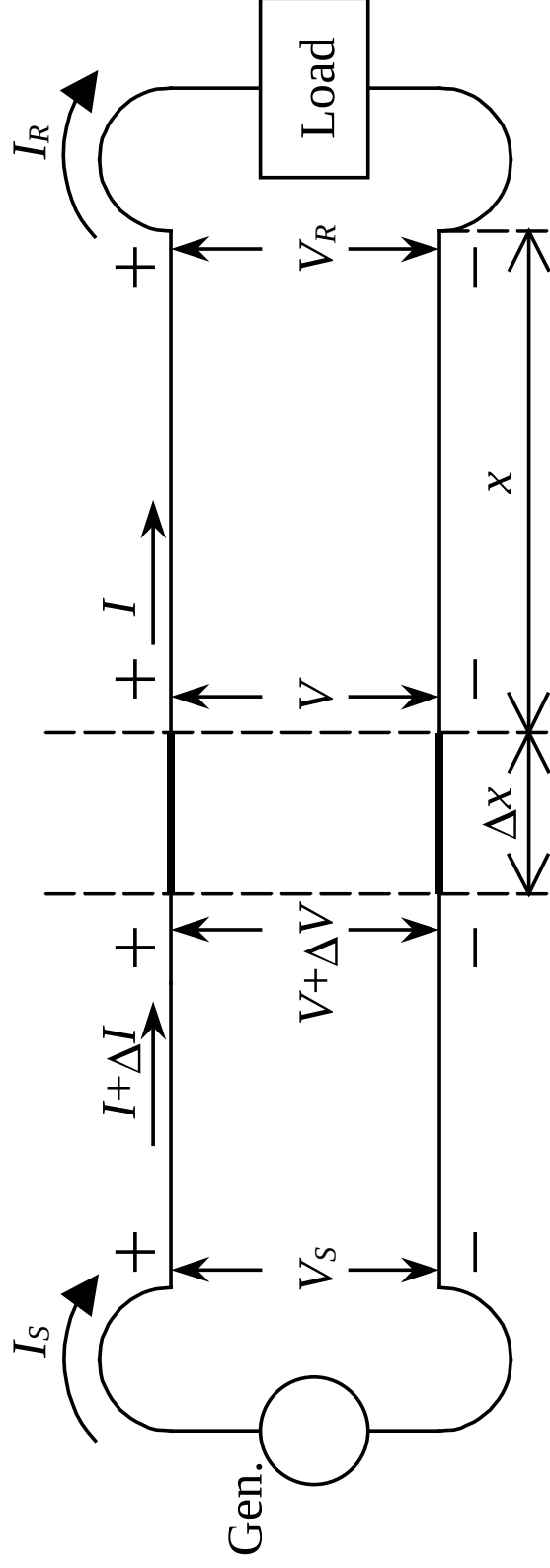


$$V(x + \Delta x) = V(x) + z \Delta x I(x) \quad I(x + \Delta x) = I(x) + y \Delta x V(x + \Delta x)$$

$$\frac{V(x + \Delta x) - V(x)}{\Delta x} = z I(x) \quad \frac{I(x + \Delta x) - I(x)}{\Delta x} = y V(x + \Delta x)$$

$$\lim_{\Delta x \rightarrow 0} \frac{dV(x)}{dx} = z I(x) \quad \lim_{\Delta x \rightarrow 0} \frac{dI(x)}{dx} = y V(x)$$

Long Transmission Line Model



$$\frac{d^2 V(x)}{dx^2} = z \frac{dI(x)}{dx} \quad \frac{d^2 I(x)}{dx^2} = y \frac{dV(x)}{dx}$$

$$\frac{d^2 V(x)}{dx^2} = z (y V(x)) \quad \frac{d^2 I(x)}{dx^2} = y (z I(x))$$

Power Systems I $\gamma^2 = z y$ propagation constant

Long Transmission Line Model

$$\frac{d^2V(x)}{dx^2} = \gamma^2 V(x)$$

$$V = A_1 e^{\gamma x} + A_2 e^{-\gamma x}$$

$$\gamma = \alpha + j\beta = \sqrt{zy} = \sqrt{(r + j\omega L)(g + j\omega C)}$$

$$I(x) = \frac{1}{z} \frac{dV(x)}{dx} = \frac{\gamma}{z} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) = \sqrt{\frac{y}{z}} (A_1 e^{\gamma x} - A_2 e^{-\gamma x})$$

$$Z_c = \sqrt{z/y} \quad I(x) = \frac{1}{Z_c} (A_1 e^{\gamma x} - A_2 e^{-\gamma x}) \quad \text{characteristic impedance}$$

$$@ x = 0 \Rightarrow \quad A_1 = \frac{V_R + I_R Z_c}{2} \quad A_2 = \frac{V_R - I_R Z_c}{2}$$

Power Systems I

Long Transmission Line Model

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{x\sqrt{yz}} + \frac{V_R - Z_c I_R}{2} e^{-x\sqrt{yz}}$$

$$I(x) = \frac{V_R/Z_c + I_R}{2} e^{x\sqrt{yz}} - \frac{V_R/Z_c - I_R}{2} e^{-x\sqrt{yz}}$$

$$V(x) = \frac{e^{x\sqrt{yz}} + e^{-x\sqrt{yz}}}{2} V_R + Z_c \frac{e^{x\sqrt{yz}} - e^{-x\sqrt{yz}}}{2} I_R$$

$$I(x) = \frac{1}{Z_c} \frac{e^{x\sqrt{yz}} - e^{-x\sqrt{yz}}}{2} V_R + \frac{e^{x\sqrt{yz}} + e^{-x\sqrt{yz}}}{2} I_R$$

$$V(x) = \cosh(x\sqrt{yz}) V_R + Z_c \sinh(x\sqrt{yz}) I_R$$

$$I(x) = \frac{1}{Z_c} \sinh(x\sqrt{yz}) V_R + \cosh(x\sqrt{yz}) I_R$$

Power Systems I

Hyperbolic Functions

$$\sinh \theta = \frac{e^{\theta} - e^{-\theta}}{2}$$

$$\cosh \theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

Two-Port Representation

let $x \rightarrow \ell$

$$V_S = \cosh(\gamma \ell) V_R + Z_c \sinh(\gamma \ell) I_R$$

$$I_S = \frac{1}{Z_c} \sinh(\gamma \ell) V_R + \cosh(\gamma \ell) I_R$$

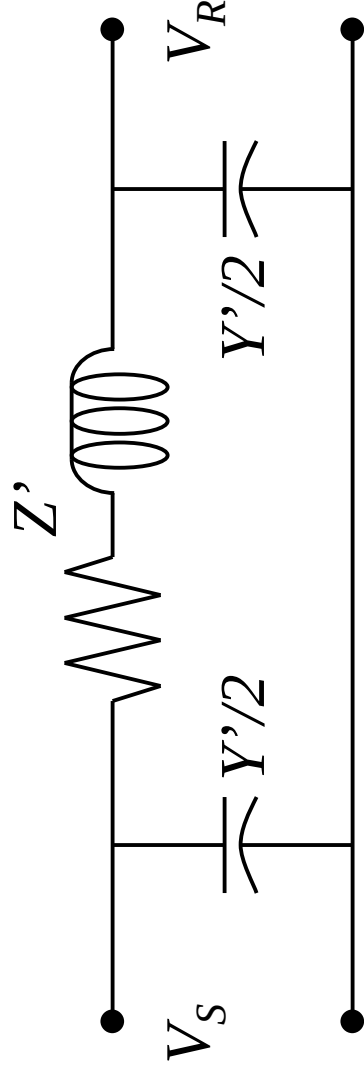
$$ABCD = \begin{bmatrix} \cosh(\gamma \ell) & Z_c \sinh(\gamma \ell) \\ \frac{1}{Z_c} \sinh(\gamma \ell) & \cosh(\gamma \ell) \end{bmatrix}$$

$$\gamma = \sqrt{z} \, y \qquad Z_c = \sqrt{\frac{z}{y}}$$

Pi-Model of a Long Transmission Line

- Represent a long transmission line as a pi-model for circuit analysis

- The circuit:



- Find the values for Z' and Y'

$$V_S = \left(1 + \frac{Z'Y'}{2}\right)V_R + Z'I_R \quad \rightarrow Z' = Z_c \sinh(\gamma \ell)$$

$$I_S = Y'\left(1 + \frac{Z'Y'}{4}\right)V_R + \left(1 + \frac{Z'Y'}{2}\right)I_R \quad \rightarrow$$

$$\frac{Y'}{2} = \frac{1}{Z'}(\cosh(\gamma \ell) - 1) = \frac{\cosh(\gamma \ell) - 1}{Z_c \sinh(\gamma \ell)} = \frac{1}{Z_c} \tanh\left(\frac{\gamma \ell}{2}\right)$$

Long Transmission Line Example

- 250 km, 500 kV transmission line has per phase
 - ◆ $z = 0.045 + j 0.4 \Omega/\text{km}$ $Y = j 4.0 \mu\text{S}/\text{km}$
- Find ABCD for a pi model of the long transmission line

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{0.045 + j0.4}{4 \times 10^{-6}}} = 316.7 - j17.76$$

$$\gamma = \sqrt{zy} = \sqrt{(0.045 + j0.4)(4 \times 10^{-6})} = 7.104 \times 10^{-5} + j0.001267$$

$$Z' = Z_c \sinh(\gamma \ell) = 10.88 + j98.36$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh\left(\frac{\gamma \ell}{2}\right) = j0.001008$$

Long Transmission Line Example

$$Z' = 10.88 + j98.36$$

$$\frac{Y'}{2} = j0.001008$$

$$A = D = \left(1 + \frac{Z'Y'}{2}\right) = 0.9504 + j0.0055$$

$$B = Z' = 10.88 + j98.36$$

$$C = Y' \left(1 + \frac{Z'Y'}{4}\right) = j0.00100$$