

Power Flow Solution

- **The utility wants to know the voltage profile**
 - ◆ the nodal voltages for a given load and generation schedule
- **Types of network buses**
 - ◆ load bus
 - known real (P) and reactive (Q) power injections
 - ◆ generator bus
 - known real (P) power injection and the voltage magnitude (V)
 - ◆ slack bus (swing bus)
 - known voltage magnitude (V) and voltage angle (δ)
 - must have one generator as the slack bus
 - takes up the power slack due to losses in the network

Power Flow Equations

- KCL

$$\begin{aligned} I_i &= y_{i0}V_i + y_{i1}(V_i - V_1) + y_{i2}(V_i - V_2) + \dots + y_{in}(V_i - V_n) \\ &= (y_{i0} + y_{i1} + y_{i2} + \dots + y_{in})V_i - y_{i1}V_1 - y_{i2}V_2 - \dots - y_{in}V_n \\ &= V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij}V_j \quad j \neq i \end{aligned}$$

- Power Law

$$P_i + jQ_i = V_i I_i^* \quad \rightarrow \quad I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij}V_j \quad j \neq i$$

Power Systems I

Gauss-Seidel Method

- **A non-linear algebraic equation solver**
 - ♦ method of successive displacements
 - ♦ iterative steps:
 - take a function and rearrange it into the form $x = g(x)$
{there are several possible arrangements}
 - make an initial estimate of the variable x : $x^{[0]}$ = initial value
 - find an iterative improvement of $x^{[k]}$, that is: $x^{[k+1]} = g(x^{[k]})$
 - a solution is reached when the difference between two iterations is less than a specified accuracy: $|x^{[k+1]} - x^{[k]}| \leq \varepsilon$
 - ♦ acceleration factors
 - can improve the rate of convergence: $\alpha > 1$
 - modified step: the improvement is found as
 $x^{[k+1]} = x^{[k]} + \alpha(g(x^{[k]}) - x^{[k]})$

Gauss-Seidel Example

- Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$
 - ♦ Step 1. Cast the equation into the $g(x)$ form.

$$9x = -x^3 + 6x^2 + 4$$

$$x = -\frac{1}{9}x^3 + \frac{6}{9}x^2 + \frac{4}{9} = g(x)$$

Gauss-Seidel Example

- ◆ Step 2. Starting with an initial guess of $x[0] = 2$, several iterations are performed.

$$x^{[1]} = g(x^{[0]} = 2) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222$$

$$x^{[2]} = g(x^{[1]} = 2.2222) = -\frac{1}{9}(2.2222)^3 + \frac{6}{9}(2.2222)^2 + \frac{4}{9} = 2.5173$$

$$x^{[3]} = g(x^{[2]} = 2.5173) = -\frac{1}{9}(2.5173)^3 + \frac{6}{9}(2.5173)^2 + \frac{4}{9} = 2.8966$$

$$x^{[4]} = 3.3376$$

$$x^{[5]} = 3.7398$$

$$x^{[6]} = 3.9568$$

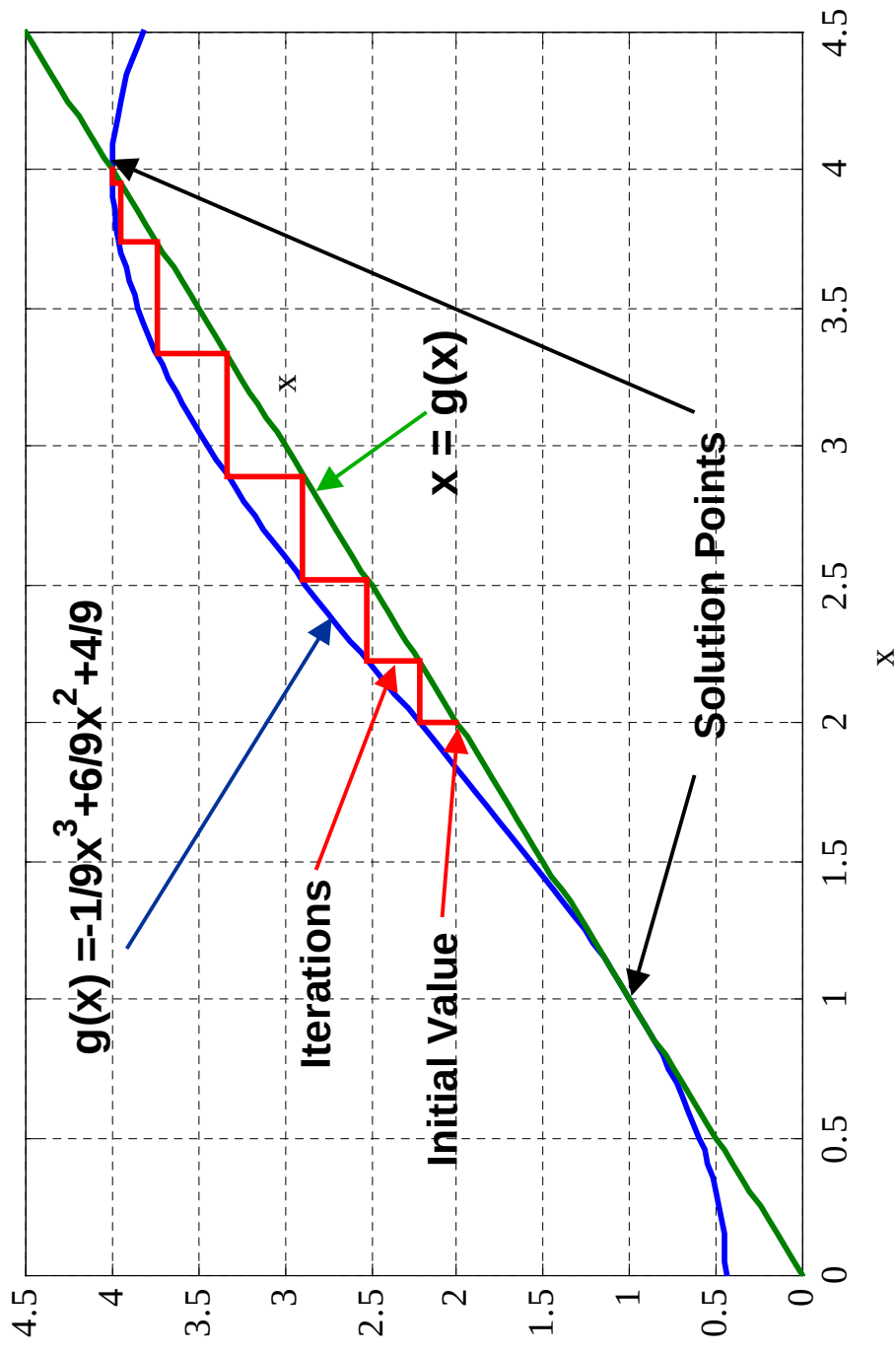
$$x^{[7]} = 3.9988$$

$$x^{[8]} = 4.0000$$

Power Systems I

Gauss-Seidel Example

Matlab Results



Gauss-Seidel Example

- Find the root of the equation: $f(x) = x^3 - 6x^2 + 9x - 4 = 0$ with an acceleration factor of 1.25
- ◆ Starting with an initial guess of $x^{[0]} = 2$.

$$x^{[0]} = 2$$

$$g(2) = -\frac{1}{9}(2)^3 + \frac{6}{9}(2)^2 + \frac{4}{9} = 2.2222$$

$$x^{[1]} = 2 + 1.25[2.2222 - 2] = 2.2778$$

$$g(2.2778) = -\frac{1}{9}(2.2778)^3 + \frac{6}{9}(2.2778)^2 + \frac{4}{9} = 2.5902$$

$$x^{[2]} = 2.2778 + 1.25[2.5902 - 2.2778] = 2.6683$$

Gauss-Seidel Example

- ◆ Additional iterations

$$x^{[3]} = 3.0801$$

$$x^{[4]} = 3.1831$$

$$x^{[5]} = 3.7238$$

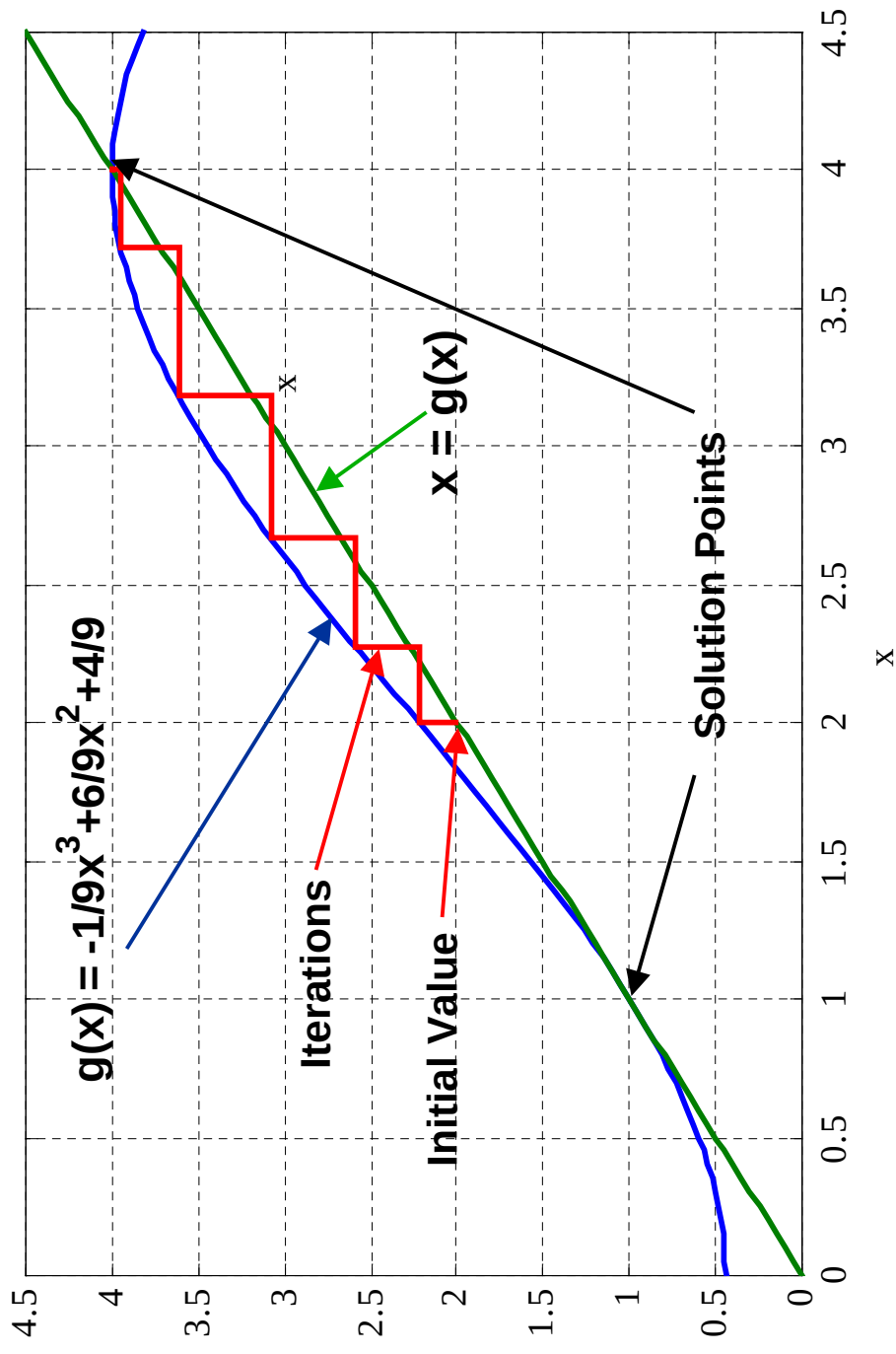
$$x^{[6]} = 4.0084$$

$$x^{[7]} = 3.9978$$

$$x^{[8]} = 4.0005$$

Gauss-Seidel Example

Matlab Results



Power Systems I

with acceleration factor: 1.25

Gauss-Seidel for a System of Equations

- Consider a system of n equations

$$f_1(x_1, x_2, \dots, x_n) = C_1$$

$$f_2(x_1, x_2, \dots, x_n) = C_2$$

$$\vdots$$

$$f_n(x_1, x_2, \dots, x_n) = C_n$$

- Rearrange each equation for one of the variables

$$x_1 = C_1 + g_1(x_1, x_2, \dots, x_n)$$

$$x_2 = C_2 + g_2(x_1, x_2, \dots, x_n)$$

$$\vdots$$

$$x_n = C_n + g_n(x_1, x_2, \dots, x_n)$$

Power Systems I

Gauss-Seidel for a System of Equations

- **steps**

- ◆ assume an approximate solution for the independent variables

$$\left(x_1^{[0]}, x_2^{[0]}, \dots, x_n^{[0]} \right)$$

- ◆ find the results in a new approximate solution

$$\left(x_1^{[k+1]}, x_2^{[k+1]}, \dots, x_n^{[k+1]} \right)$$

- ◆ in the Gauss-Seidel method, the updated values of the variables calculated in the preceding equations are used immediately in the solution of the subsequent equations

The Power Flow Equation

- The equation

$$P_i + jQ_i = V_i I_i^* \rightarrow I_i = \frac{P_i - jQ_i}{V_i^*}$$

$$\frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j \quad j \neq i$$

- The Gauss-Siedel form

$$V_i = \frac{\frac{P_i - jQ_i}{V_i^*} + \sum_{j=1}^n y_{ij} V_j}{\sum_{j=0}^n y_{ij}} \quad j \neq i \Rightarrow V_i^{[k+1]} = \frac{\frac{P_i - jQ_i}{V_i^{*[k]}} + \sum_{j=1}^n y_{ij} V_j^{[k]}}{\sum_{j=0}^n y_{ij}} \quad j \neq i$$

Power Injections

- Rewriting the power equation to find P and Q

$$P_i^{[k+1]} = \Re \left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

$$Q_i^{[k+1]} = -\Im \left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

- ♦ the real and reactive powers are scheduled for the load buses that is, they remain fixed
- ♦ the currents and powers are expressed as going into the bus
 - for generation the powers are positive
 - for loads the powers are negative
 - the scheduled power is the sum of the generation and load powers

Power Systems I

Solution by Gauss-Seidel

- The complete set of equations become:

$$V_i^{[k+1]} = \frac{P_i^{[sch]} - jQ_i^{[sch]} + \sum_{j=1}^n y_{ij} V_j^{[k]}}{\sum_{j=0}^n y_{ij}} \quad j \neq i$$

$$P_i^{[k+1]} = \Re \left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

$$Q_i^{[k+1]} = -\Im \left\{ V_i^{*[k]} \left[V_i^{[k]} \sum_{j=0}^n y_{ij} - \sum_{j=1}^n y_{ij} V_j^{[k]} \right] \right\} \quad j \neq i$$

Solution by Gauss-Seidel

- Rewriting the equations in terms of the Y-Bus

$$V_i^{[k+1]} = \frac{\frac{P_i^{[sch]} - jQ_i^{[sch]}}{V_i^{*[k]}} - \sum_{j=1, j \neq i}^n Y_{ij} V_j^{[k]}}{Y_{ii}}$$

$$P_i^{[k+1]} = \Re \left\{ V_i^{*[k]} \left[V_i^{[k]} Y_{ii} + \sum_{j=1, j \neq i}^n Y_{ij} V_j^{[k]} \right] \right\}$$

$$Q_i^{[k+1]} = -\Im \left\{ V_i^{*[k]} \left[V_i^{[k]} Y_{ii} + \sum_{j=1, j \neq i}^n Y_{ij} V_j^{[k]} \right] \right\}$$

Power Systems I

Solution by Gauss-Seidel

- **System characteristics**

- ◆ Since both components (V & δ) are specified for the slack bus, there are $2(n - 1)$ equations which must be solved iteratively
- ◆ For the load buses, the real and reactive powers are known: scheduled
 - the voltage magnitude and angle must be estimated
 - in per unit, the nominal voltage magnitude is 1 pu
 - the angles are generally close together, so an initial value of 0 degrees is appropriate

Solution by Gauss-Seidel

- ◆ For the generator buses, the real power and voltage magnitude are known
 - the real power is scheduled
 - the reactive power is computed based on the estimated voltage values
 - the voltage is computed by Gauss-Seidel, only the imaginary part is kept
 - the complex voltage is found from the magnitude and the iterative imaginary part

$$e_i^{[k+1]} = \sqrt{|V_i^{[sch]}|^2 - (f_i^{[k+1]})^2} \quad V_i = e_i + j f_i$$

Example

- Using the Gauss-Seidel method, determine the phasor values of the voltage at the load buses 2 and 3, accurate to 2 decimal places

